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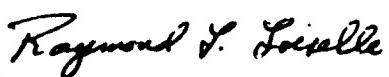
Raymond L. Loiselle, Lt.Col., USAF
Chief, ESD Lincoln Laboratory Project Office

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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# ANALYSIS OF DIRECT MECHANICAL STABILIZATION PLATFORM
## FOR ANTENNA STABILIZATION
## AND CONTROL IN A SHIPBOARD ENVIRONMENT

*M. TAYLOR*

*Group 76*

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Abstract

This report presents a rigid body analysis of the dynamics of a shipboard antenna in a three-axis gimbal configuration utilizing Direct Mechanical Stabilization (DMS) for primary control. System parameters are defined and Euler equations applied to obtain a non-linear system model that includes expected torque inputs. A computer simulation is generated with ship input motions due to roll, pitch, turn and flexure and a numerical integration procedure is then employed to determine the motion of the antenna and the resulting pointing error for various scenarios.

The results of the cases studied indicate potential feasibility of this approach for applications requiring pointing accuracy of approximately ±0.15 degree.

# CONTENTS

## APPENDICES

# LIST OF ILLUSTRATIONS

## 1.0  INTRODUCTION

This is the rigid body analysis of a DMS*-controlled antenna on board a surface ship for a three-axis gimbal configuration employing X-Y over train.  The combined effects of roll, pitch, turn maneuvers and ship flexure on the pointing accuracy of the antenna are studied.

### 1.1  Background

The gyroscope has found its most prolific application in modern technology as an angular direction and motion sensor whose electrical output signal is used to activate either a mechanical or computational device.  However, throughout the last half century of very active development and refinement of these sensors of shrinking size, there has been a lower level but continuing effort on large gyro wheels for DMS.  The most sophisticated effort on DMS in recent years has been associated with the use of control moment gyros for stabilization and attitude control of satellites.

There are two general approaches to the concept of DMS:

1)  Mount the stabilized device directly to the frame carrying the gyro rotor.  This assembly is then mounted within gimbals and bearings to permit angular isolation from input motion. Examples of this approach are found in both early and more recent literature.  Scarborough[1] describes the gyro pendulum; Williams[2] analyzes stabilization of an optical sensor for a shipboard application; Bieser[4] describes a configuration of this type for stabilizing an antenna on board a ship.

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* Direct Mechanical Stabilization

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In this general approach the gyro spin axis and the stabilized device (optics, antenna, etc.) move together as the gyro precesses in response to disturbing torques.

2)   In the second general approach the gyro rotor spin assembly is mounted to the stabilized frame within a pivot axis that is orthogonal to the spin axis. This is a distinct improvement in stabilizing effectiveness over the first type described in 1) above. Now the primary effect of any disturbing torque on the stabilized device is to cause only the gyro to precess within its pivot axis. The stabilized frame remains essentially undisturbed while the gyro precesses. This method has been used since the beginning of the century on examples such as the Schleck ship stabilizer in 1903[1] and the monorail stabilizer of Scherl and Schilorsky in 1909[1]. More recent examples are Bieser's antenna stabilizer[3], Matthews version of a similar device[5] and an aircraft camera stabilizer development by Westinghouse Canada Limited[6].

The dynamics of the Bieser approach[3] have recently been analyzed[8]. This configuration utilizes the DMS principle to provide a roll-pitch stabilized platform carrying a train-elevation servo-controlled antenna.

The X-Y configuration described in this report applies the second general approach to directly position and stabilize the antenna without an intermediate train-elevation servo.

## 1.2  General Description

Figure 1 is a pictorial representation of the hardware configuration showing the antenna in a horizontal beam position, mounted to the stabilized structure containing the 'A' and 'B' DMS gyro assemblies. The first gimbal axis supporting the stabilized assembly is the Y axis which is nominally controlled

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Y-AXIS

ANTENNA

A-GYRO

B-GYRO

X-AXIS

TRAIN PICKOFF

TRAIN STEPPER MOTOR
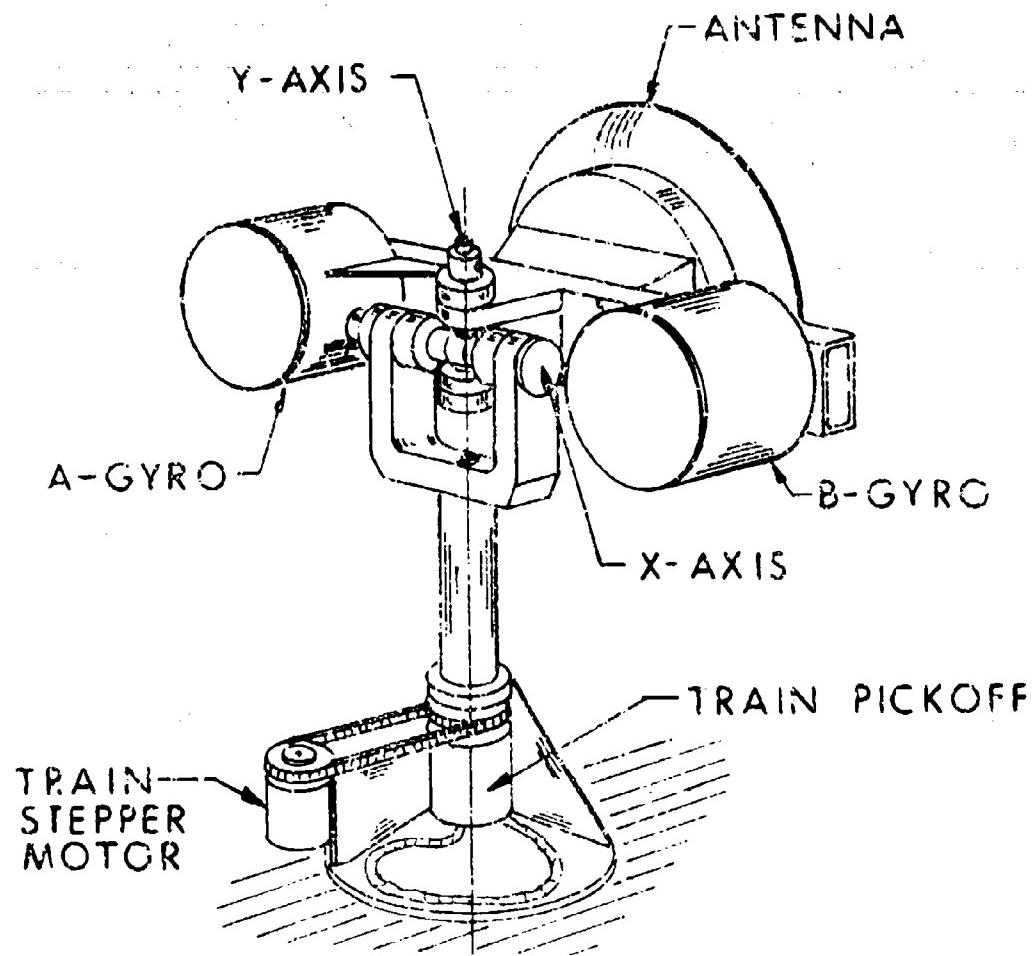
Fig. 1.   X-Y DMS pedestal picture.

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to be in the vertical plane containing the satellite; the next supporting axis is the X axis which is nominally controlled to be perpendicular to the vertical plane containing the satellite. The total assembly is carried on the train axis which is mounted to the deck of the ship.

Further detail is given in Figure 2 where the antenna is shown in a vertical beam position. The 'A' gyro is free to pivot about an axis parallel to X and the 'B' gyro is free to pivot about an axis parallel to Y. The gyro pivot axes and the X, Y gimbal axes are each equipped with a torquer and angular pickoff. The train axis is positioned by a stepper motor, and has a precise angular pickoff to read its position.

Antenna beam position control is achieved by applying appropriate control torques to the 'A' or 'B' gyro torquer. For example, antenna motion about the X axis is achieved by applying a torque to the 'B' gyro torquer. The antenna is stabilized against the effects of disturbing torques by precession of the gyro assemblies within their respective pivot axes. Narrow bandwidth caging loops are used to maintain the spin axes nominally parallel to the antenna beam axis. For example, the X torquer responds to a pickoff signal from the 'B' gyro to null the 'B' gyro pickoff output.

The details of the control concepts will be explained in the body of this report.
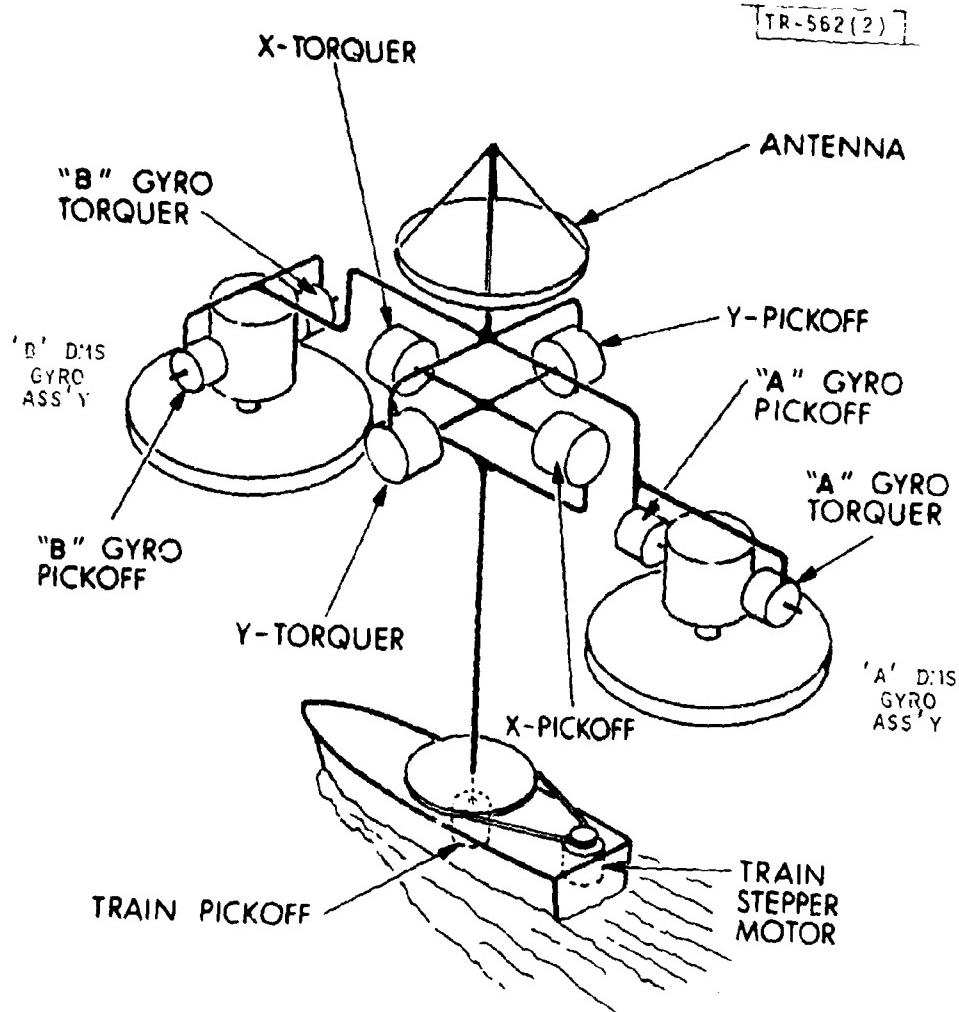
Fig. 2.   X-Y DMS components and structure.

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## 2.0 KINEMATIC FRAMES & EULER ROTATIONS

Referring to Figure 3, the following reference frames and their Euler rotations are defined:

$\{\hat{\underline{n}}\}$ = inertial frame

$\{\hat{\underline{f}}\}$ = deck frame

$\{\hat{\underline{p}}\}$ = antenna platform

$\{\hat{\underline{a}}\}$ = 'A' gyro gimbal

$\{\hat{\underline{b}}\}$ = 'B' gyro gimbal

$\{\hat{\underline{e}}\}$ = frame containing satellite position along $\hat{\underline{e}}_2$

$\{\hat{\underline{g}}\}$ = X-Y gimbal frame

Additional intermediate reference frames are defined below in terms of the angles of rotation and the axis of each rotation.

$\psi_A$ = ship azimuth

$\psi_P$ = pitch

$\psi_R$ = roll

$\psi_H$ = heel $\quad (\psi_{RH} = \psi_R + \psi_H)$

$\theta_T$ = train

$\theta_X$ = X axis

$\theta_Y$ = Y axis

$\gamma_a$ = gyro 'A' gimbal rotation with respect to platform

$\gamma_b$ = gyro 'B' gimbal rotation with respect to platform

$\alpha_A$ = satellite azimuth position

$\alpha_E$ = satellite elevation position

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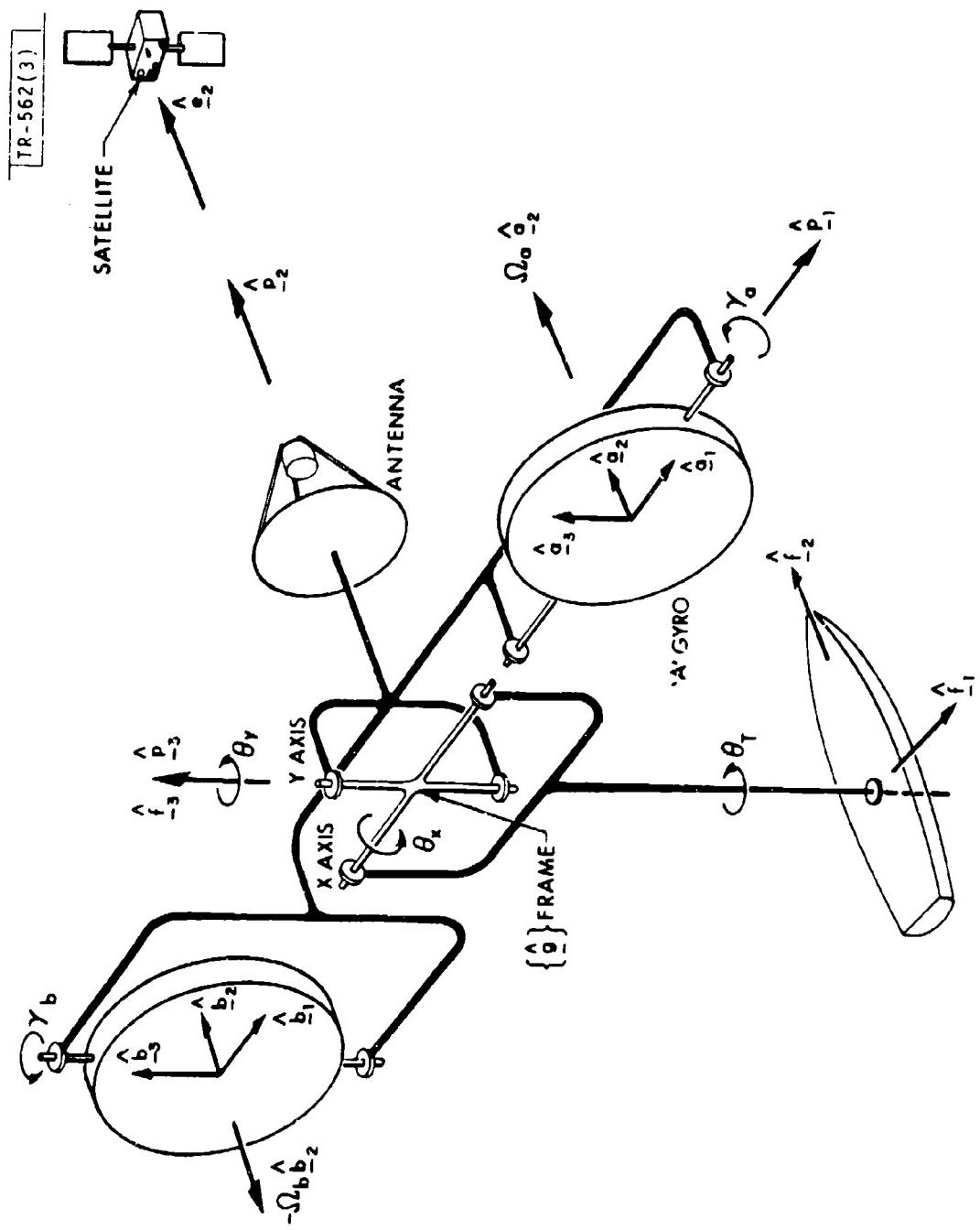
Fig. 3.   X-Y DMS kinematic frames.

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The format for showing the rotation sequences are:

$$\begin{Bmatrix} \text{starting} \\ \text{frame} \end{Bmatrix} \quad \xrightarrow[\text{Rotation Axis}]{\text{Rotation Angle}} \quad \begin{Bmatrix} \text{frame after} \\ \text{rotation} \end{Bmatrix}$$
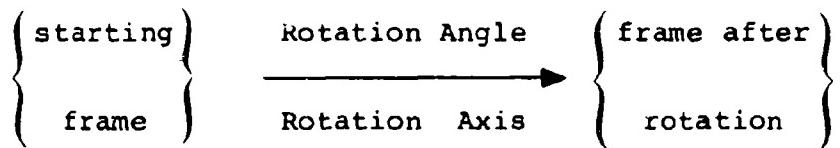
Applying this format and the previously defined frames and rotation angles the rotation sequences are as follows:

<u>Deck of Ship</u> $\{\hat{\underline{f}}\}$

$$\{\hat{\underline{n}}\} \quad \xrightarrow[\hat{\underline{n}}_3]{\psi_A} \quad \{\hat{\underline{n}}'\} \quad \xrightarrow[\hat{\underline{n}}'_1]{\psi_P} \quad \{\hat{\underline{n}}''\} \quad \xrightarrow[\hat{\underline{n}}''_2]{\psi_{RH}} \quad \{\hat{\underline{f}}\}$$

<u>Antenna Platform</u> $\{\hat{\underline{p}}\}$

$$\{\hat{\underline{f}}\} \quad \xrightarrow[\hat{\underline{f}}_3]{\theta_T} \quad \{\hat{\underline{f}}'\} \quad \xrightarrow[\hat{\underline{f}}'_1]{\theta_X} \quad \{\hat{\underline{g}}\} \quad \xrightarrow[\hat{\underline{g}}_3]{\theta_Y} \quad \{\hat{\underline{p}}\}$$

<u>'A' Gyro</u> $\{\hat{\underline{a}}\}$

$$\{\hat{\underline{p}}\} \quad \xrightarrow[\hat{\underline{p}}_1]{\gamma_a} \quad \{\hat{\underline{a}}\}$$

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'B' Gyro $\{\hat{\underline{b}}\}$

$$\{\hat{\underline{p}}\} \quad \xrightarrow[\hat{\underline{p}}_3]{\gamma_b} \quad \{\hat{\underline{b}}\}$$
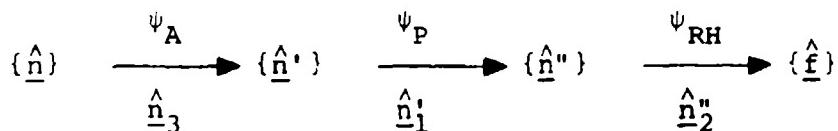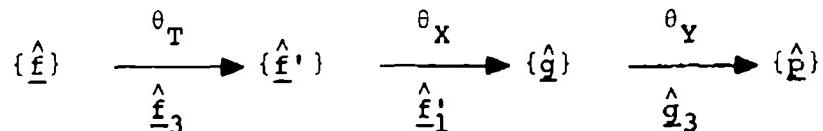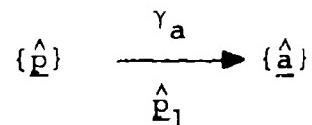
Satellite Position Vector Along $\{\hat{\underline{e}}_2\}$

$$\{\hat{\underline{n}}\} \quad \xrightarrow[\hat{\underline{n}}_3]{\alpha_A} \quad \{\hat{\underline{e}}'\} \quad \xrightarrow[\hat{\underline{e}}'_1]{\alpha_E} \quad \{\hat{\underline{e}}\}$$

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## 3.0 ANGULAR VELOCITIES

The following angular velocities are defined:

### Deck of Ship $\{\hat{\underline{f}}\}$

$$\underline{\omega}_f = \omega_{f1}\,\hat{\underline{f}}_1 + \omega_{f2}\,\hat{\underline{f}}_2 + \omega_{f3}\,\hat{\underline{f}}_3 \tag{3-1}$$

$$\underline{\omega}_f = \dot{\psi}_A\,\hat{\underline{n}}_3' + \dot{\psi}_P\,\hat{\underline{n}}_1'' + \dot{\psi}_R\,\hat{\underline{f}}_2 \tag{3-2}$$

### Antenna Platform $\{\hat{\underline{p}}\}$

$$\underline{\omega}_p = \omega_{p1}\,\hat{\underline{p}}_1 + \omega_{p2}\,\hat{\underline{p}}_2 + \omega_{p3}\,\hat{\underline{p}}_3 \tag{3-3}$$

$$\underline{\omega}_p = \underline{\omega}_f + \dot{\theta}_T\,\hat{\underline{f}}_3' + \dot{\theta}_x\,\hat{\underline{q}}_1 + \dot{\theta}_y\,\hat{\underline{p}}_3 \tag{3-4}$$

### 'A' Gyro $\{\hat{\underline{a}}\}$

Gimbal:
$$\underline{\omega}_a = \omega_{a1}\,\hat{\underline{a}}_1 + \omega_{a2}\,\hat{\underline{a}}_2 + \omega_{a3}\,\hat{\underline{a}}_3 \tag{3-5}$$

Rotor:
$$\underline{\Omega}_a = \omega_{a1}\,\hat{\underline{a}}_1 + \Omega_a\,\hat{\underline{a}}_2 + \omega_{a3}\,\hat{\underline{a}}_3 \tag{3-6}$$

where $\Omega_a$ = constant wheel spin speed

Note that $\dot{\gamma}_a = \omega_{a1} - \omega_{p1}$

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'B' Gyro $\{\hat{\underline{b}}\}$

Gimbal: $\quad \underline{\omega}_b = \omega_{b1} \, \hat{\underline{b}}_1 + \omega_{b2} \, \hat{\underline{b}}_2 + \omega_{b3} \, \hat{\underline{b}}_3$ (3-7)

Rotor: $\quad \underline{\Omega}_b = \omega_{b1} \, \hat{\underline{b}}_1 - \Omega_b \, \hat{\underline{b}}_2 + \omega_{b3} \, \hat{\underline{b}}_3$ (3-8)

Note that the polarity of $\Omega_b \, \hat{\underline{b}}_2$ is negative. This is essential to assure that both rotors precess in the same direction with the antenna when a control torque is applied to either rotor. For example the torquer used to control X axis motion applies a torque to the 'B' gyro about $\hat{\underline{b}}_3$ and, via reaction, an opposite polarity torque to the 'A' gyro about $\hat{\underline{a}}_3$. Both rotors then precess in the same direction about $\hat{\underline{b}}_1$ and $\hat{\underline{a}}_1$ respectively while the antenna moves about $\hat{\underline{p}}_1$.

Note that $\dot{\gamma}_b = \omega_{b3} - \omega_{p3}$.

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## 4.0 STATE VARIABLES

The following state variables are defined for the system equations that will be developed:

$$X_1 = \theta_x$$

$$X_2 = \theta_y$$

$$X_3 = \omega_{a1}$$

$$X_4 = \omega_{b3}$$

$$X_5 = \omega_{p3}$$

$$X_6 = \omega_{p1}$$

$$X_7 = \omega_{p2}$$

$$X_8 = \gamma_a$$

$$X_9 = \gamma_b$$

$$X_{10} = \int \gamma_a dt$$

$$X_{11} = \int \gamma_b dt$$

$$X_{12} = \theta_T$$

$$X_{13} = \dot{\theta}_T$$

## 5.0  GYRO EULER EQUATIONS

The Euler equations for the gyros are developed in Appendix A and result in the following relations:

$$\dot{\omega}_{a1} = J_{a1} + T_{a1}/I_{g1} \tag{5-1}$$

$$\dot{\omega}_{a3} = J_{a3} + T_{a3}/I_{g3} \tag{5-2}$$

$$\dot{\omega}_{b1} = J_{b1} + T_{b1}/I_{g1} \tag{5-3}$$

$$\dot{\omega}_{b3} = J_{b3} + T_{b3}/I_{g3} \tag{5-4}$$

where

$$J_{a1} = \omega_{a3}\Omega_a \, I_{g2}/I_{g1} - \omega_{a3}\omega_{a2}$$

$$J_{a3} = -\omega_{a1}\Omega_a \, I_{g2}/I_{g1} + \omega_{a1}\omega_{a2}$$

$$J_{b1} = -\omega_{b3}\Omega_b \, I_{g2}/I_{g1} - \omega_{b3}\omega_{b2}$$

$$J_{b3} = \omega_{b1}\Omega_b \, I_{g2}/I_{g1} + \omega_{b1}\omega_{b2}$$

$I_{g1}, I_{g2}$ = gyro assembly inertia about 1 and 2 axes of $\{\hat{\underline{a}}\}$ or $\{\hat{\underline{b}}\}$

$T_{a1}, T_{a3}, T_{b1}, T_{b3}$ = torque applied to gyro about subscript axis (see Eq. (9-6) and (9-12))

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## 6.0 PLATFORM EULER EQUATIONS

The platform Euler equations are developed in Appendix A
and result in the following relations:

$$\dot{\omega}_{p1} = T_{p1}/I_{p1} \qquad (6-1)$$

$$\dot{\omega}_{p2} = T_{p2}/I_{p2} \qquad (6-2)$$

$$\dot{\omega}_{p3} = T_{p3}/I_{p3} \qquad (6-3)$$

where

$$T_{pj} = \text{torque applied to platform about } j \text{ axis}$$
$$\text{(see Eq. (9-20))}$$

## 7.0 CONSTRAINTS

Mechanical constraints exist due to the gimballing config-
uration. The angular velocity equations that resolve these
constraints are developed in Appendix B with the following
results:

$$\omega_{a2} = \omega_{p2} \cos \gamma_a + \omega_{p3} \sin \gamma_a \qquad (7-1)$$

$$\omega_{a3} = -\omega_{p2} \sin \gamma_a + \omega_{p3} \cos \gamma_a \qquad (7-2)$$

$$\omega_{b1} = \omega_{p1} \cos \gamma_b + \omega_{p2} \sin \gamma_b \qquad (7-3)$$

$$\omega_{b2} = -\omega_{p1} \sin \gamma_b + \omega_{p2} \cos \gamma_b \qquad (7-4)$$

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## 8.0 TRANSFORMATIONS AND CONTROL OF $\theta_x$, $\theta_y$, $\theta_T$

### 8.1 Transformation for $\theta_x$, and $\theta_y$

The antenna beam lies along axis $\hat{\underline{p}}_2$; the satellite position vector lies along $\hat{\underline{e}}_2$. We desire to orient $\hat{\underline{p}}_2$ coincident with $\hat{\underline{e}}_2$ using the rotations defined in Section 2.0 to determine appropriate input command angles $\theta_{xc}$ and $\theta_{yc}$ as functions of $\alpha_A$, $\alpha_E$, $\psi_A$, $\psi_p$, $\psi_{RH}$, $\theta_T$. The computation process determines the $\{\hat{\underline{n}}\}$ components of $\hat{\underline{p}}_2$ and $\hat{\underline{e}}_2$, then equates these components to solve for $\theta_{xc}$ and $\theta_{yc}$.

The direction cosine matrices for the rotations in Section 2.0 are defined using the symbols,

$$[C_i(\zeta)] = \text{Direction cosine matrix for rotation}$$
$$\text{about } i^{th} \text{ axis of starting frame}$$
$$\text{through the angle } \zeta.$$

Using this notation the relations between various reference frames are defined as follows:

$$\left. \begin{array}{l} \{\hat{\underline{f}}'\} = [C_3(\theta_T)][C_2(\psi_{RH})][C_1(\psi_p)][C_3(\psi_A)]\{\hat{\underline{n}}\} \\[2ex] \{\hat{\underline{f}}'\} = [B_1]\{\hat{\underline{n}}\} \end{array} \right\} \qquad (8\text{-}1)$$

$$\left. \begin{array}{l} \{\hat{\underline{p}}\} = [C_3(\theta_{yc})][C_1(\theta_{xc})]\{\hat{\underline{f}}'\} \\[2ex] \{\hat{\underline{p}}\} = [B_2]\{\hat{\underline{f}}'\} \end{array} \right\} \qquad (8\text{-}2)$$

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$$\{\hat{\underline{a}}\} = [C_1(\gamma_a)]\{\hat{\underline{p}}\} \tag{8-3}$$

$$\{\hat{\underline{b}}\} = [C_3(\gamma_b)]\{\hat{\underline{p}}\} \tag{8-4}$$

$$\{\hat{\underline{e}}\} = [C_1(\alpha_E)][C_3(\alpha_A)]\{\hat{\underline{n}}\} \tag{8-5}$$

$$\{\hat{\underline{e}}\} = [B_3]\{\hat{\underline{n}}\}$$

From (8-1) and (8-2)

$$\{\hat{\underline{p}}\} = [B_2 B_1]\{\hat{\underline{n}}\} \tag{8-6}$$

$$\{\hat{\underline{n}}\} = B_1^T B_2^T \{\hat{\underline{p}}\}$$

From (8-5)

$$\{\hat{\underline{n}}\} = B_3^T \{\hat{\underline{e}}\} \tag{8-7}$$

The $\hat{\underline{n}}$ components of $\hat{\underline{p}}_2$ and $\hat{\underline{e}}_2$ are now equated using (8-6) and (8-7)

$$B_1^T B_2^T \left\{ \begin{array}{c} 0 \\ 1 \\ 0 \end{array} \right\} = B_3^T \left\{ \begin{array}{c} 0 \\ 1 \\ 0 \end{array} \right\} \tag{8-8}$$

The expansion and solution of Eq. (8-8) for the desired angles is performed in Appendix C with the following results:

$$\theta_{yc} = \sin^{-1}(-t_1)$$ (8-9)

$$\theta_{xc} = \sin^{-1}(t_3/\cos\theta_{yc})$$ (8-10)

where

$t_1$, $t_3$ = results of matrix equation (C-3) in
Appendix C.

## 8.2 System Control

The control approach that will be used to achieve the desired values $\theta_{xc}$, $\theta_{yc}$ is shown in Figures 4 and 5.

Referring to Figure 4, the command values $\theta_{xc}$, $\theta_{yc}$ are compared to actual values measured by the X, Y pickoffs and the error signals drive the appropriate gyro torquers. A train command based on ship's heading and satellite position (see Figure 5 and Section 8.3) drives the train stepper motor. The bottom loop shown in Figure 4 is a 'housekeeping' loop that keeps the gyro spin axis nominally parallel to the antenna beam axis.

Figure 5 shows the flow of information used to generate the control commands. The ship's master compass and heading reference provide roll, pitch and heading information used in the coordinate transformations. The heading output of the compass is also utilized directly to position the train servo.

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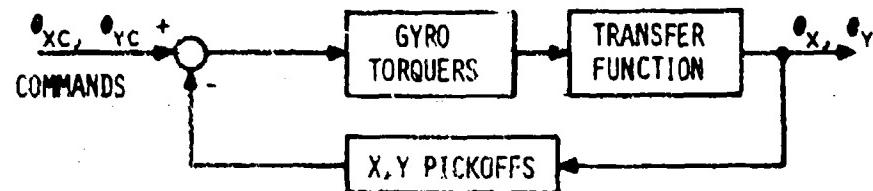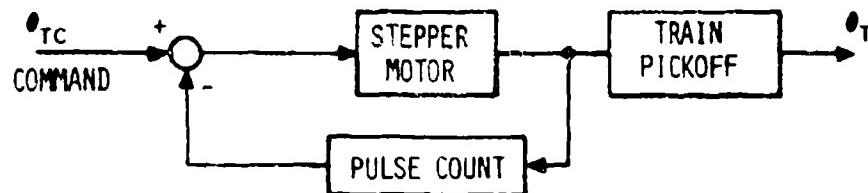
X,Y POSITION LOOPS



TRAIN POSITION LOOP
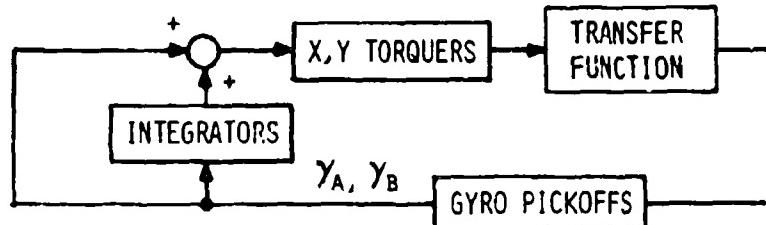


GYRO CAGING LOOPS

NOTE: THERE IS NO ACTIVE STABILIZATION LOOP. THE GYROS
PROVIDE THIS FUNCTION.

Fig. 4.   X-Y DMS control outline.

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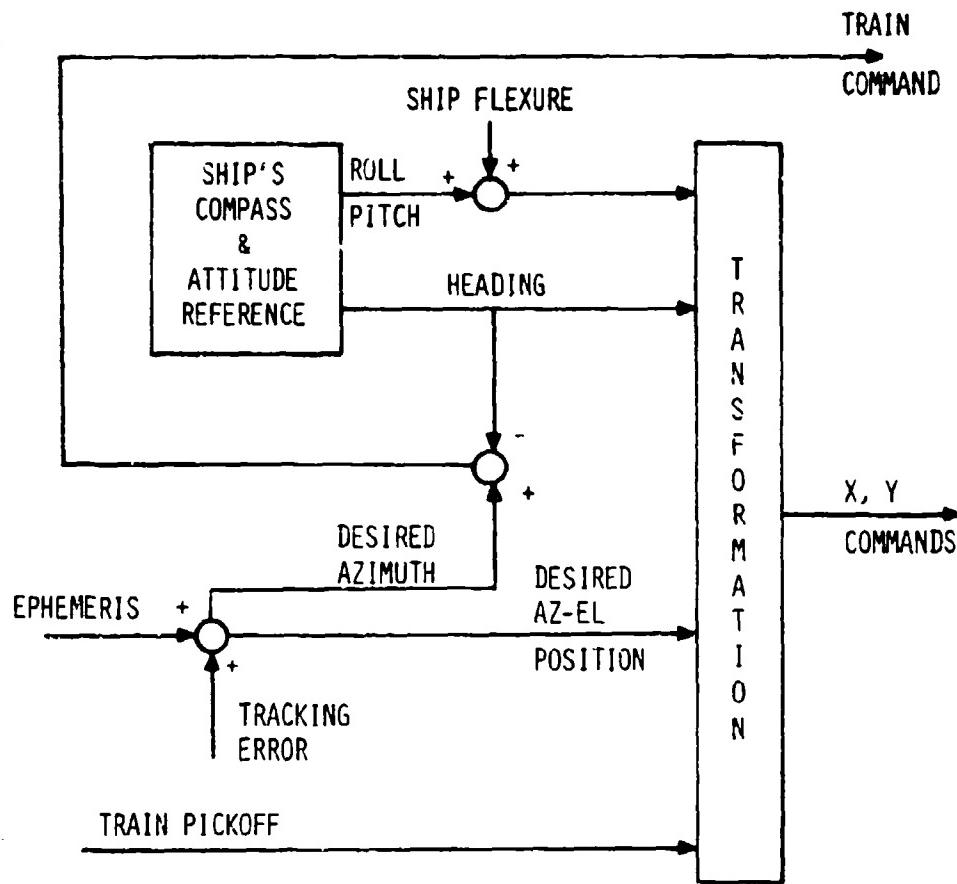
Fig. 5.   X-Y DMS commands.

The satellite ephemeris data is updated by down-link tracking error signals to provide the desired antenna beam position to the satellite. The coordinate transformation computation generates the desired values $\theta_{xc}$, $\theta_{yc}$ based on the inputs shown.

The basic stability of the DMS gyros permits a narrow bandwidth control from the command signals to the gyro torquers. This attenuates or filters the effect of high frequency ship flexure on the controlled output. Long term flexure due to thermal effects or loading are corrected by the down-link tracking error.

### 8.3 Train Axis Control of $\theta_T$

The purpose of the train axis is to nominally position the Y axis in the vertical plane containing the satellite. This is not a critical requirement and so it is expected that an inexpensive stepper motor will be used in train. For the purpose of modelling, the following damped second order system has been assumed in train:

$$\ddot{\beta} + 2\omega_n \zeta \dot{\beta} + \omega_n^2 \beta = \omega_n^2 \alpha_A \qquad (8\text{-}11)$$

where

$\beta \quad = \psi_A + \theta_T$

$\omega_n$ = natural frequency of train servo

$\zeta$ = damping factor

$\alpha_A$ = azimuth position of satellite

Making the substitution for $\beta$ as indicated above leads to the defining equation for $\theta_T$.

$$\ddot{\theta}_T + 2\omega_n\zeta\,\dot{\theta}_T + \omega_n^2\theta_T = F \tag{8-12}$$

where

$$F = -\ddot{\psi}_A - 2\omega_n\zeta\dot{\psi}_A + \omega_n^2(\alpha_A - \psi_A)$$

## 9.0  TORQUE EQUATIONS

In this section the torque contributors that produce the system response are defined for computational purposes:

### 9.1  Gyros 'A' and 'B' Torques

Spring

Friction

Unbalance

Bias

Control

Bearing Reaction

Total

<u>Platform Torques</u>

       Spring

       Friction

       Unbalance

       Bias

       Gyro Reaction

       Bearing Reaction

       Control

       Total

## 9.2  Gyro 'A' Torques

### 9.2.1  Spring

An elastic spring torque due to rotation of the 'A' gyro about its pivot axis, $\hat{\underline{a}}_1$ is modelled.

$$\underline{T}_{as} = -K_{sG} \gamma_a \hat{\underline{a}}_1 \qquad (9\text{-}1)$$

where

$\quad K_{sG}$ = gyro spring gradient

### 9.2.2  Friction

Friction about the pivot axis is modelled as a constant magnitude torque with a polarity that opposes the relative angular velocity of the 'A' gyro with respect to the platform.

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$$\underline{T}_{aF} = -K_{FG}[\dot{\gamma}_a/|\dot{\gamma}_a|]\,\hat{\underline{a}}_1 \qquad\qquad (9\text{-}2)$$

where

$\qquad\qquad K_{FG}$ = gyro friction torque amplitude

### 9.2.3 Unbalance

Torque about the pivot axis due to unbalance has been modelled as a displacement along the rotor spin axis, $\hat{\underline{a}}_2$, coupled with transverse acceleration components. Unbalance torque due to radial displacement along $\hat{\underline{a}}_3$ has been assumed to be insignificant in recognition of the fact that radial stability of balance is much better than axial stability. See Appendix D (Eq. (D-14)) for development of this expression.

$$\underline{T}_{au} = -m\ell_g[-A_2\sin\gamma_a + A_3\cos\gamma_a]\,\hat{\underline{a}}_1 \qquad\qquad (9\text{-}3)$$

where $\quad m \quad$ = gyro mass

$\qquad\ell_g \quad$ = displacement of gyro mass center from center of support along $\hat{\underline{a}}_2$

$\qquad A_2, A_3$ = acceleration components along $\hat{\underline{p}}_2$ and $\hat{\underline{p}}_3$ respectively

### 9.2.4 Bias

Bias torque has been modelled about the pivot axis.

$$\underline{T}_{aB} = T_{aB}\,\hat{\underline{a}}_1 \qquad\qquad (9\text{-}4)$$

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where

$$T_{aB} = \text{bias torque value}$$

### 9.2.5 Control

Referring to Figure 3, the control torque on the 'A' gyro is exerted about the pivot axis $\hat{\underline{a}}_1$ to correct for Y axis error as indicated by the difference between the command value, $\theta_{yc}$, and the actual value, $\theta_y$. Recall from Figure 5 that the command angle computation is degraded by ship flexure between the master attitude reference and the deck at the antenna. A conservative worst-case situation is modelled with the full ship flexure amplitude introduced into the control equation.

$$\underline{T}_{ca} = -K_a (\theta_{yc} - \theta_y + \theta_F) \, \hat{\underline{a}}_1 \qquad (9-5)$$

where

$$K_a = \text{control gain (in-lb/rad)}$$

$$\theta_F = \text{ship flexure amplitude (rad)}$$

### 9.2.6 Bearing Reaction

Radial forces at the pivot bearings introduce torque about $\hat{\underline{a}}_2$ and $\hat{\underline{a}}_3$. The component about $\hat{\underline{a}}_2$ has no significant effect since $\hat{\underline{a}}_2$ is along the spin axis of the rotor. Accordingly, the torque due to bearing reaction is defined as:

$$\underline{T}_{Ra} = T_{a3} \, \hat{\underline{a}}_3 \qquad (9-6)$$

where

$T_{a3}$ = magnitude of bearing reaction torque (in-lb)

### 9.2.7 Total Torque on 'A' Gyro

The total torque on the 'A' gyro is

$$\underline{T}_a = T_{a1} \, \hat{\underline{a}}_1 + T_{a2} \, \hat{\underline{a}}_2 + T_{a3} \, \hat{\underline{a}}_3 \qquad (9-7)$$

where

$$T_{a1} \, \hat{\underline{a}}_1 = \underline{T}_{as} + \underline{T}_{aF} + \underline{T}_{au} + \underline{T}_{aB} + \underline{T}_{cA}$$

$$T_{a2} = 0$$

### 9.3 Gyro 'B' Torques

The torques acting on the 'B' gyro are similar to those on the 'A' except for different axes and a geometric consideration applied to the control torque.

#### 9.3.1 Spring

$$\underline{T}_{bs} = -K_{sG} \gamma_b \, \hat{\underline{b}}_3 \qquad (9-8)$$

#### 9.3.2 Friction

$$\underline{T}_{bF} = -K_{FG} [\dot{\gamma}_b / |\dot{\gamma}_b|] \, \hat{\underline{b}}_3 \qquad (9-9)$$

### 9.3.3 Unbalance

$$\underline{T}_{bu} = -m\ell_g [A_1 \cos\gamma_b + A_2 \sin\gamma_b] \qquad\qquad (9\text{-}10)$$

See Eq. (D-19) in Appendix D.

### 9.3.4 Bias

$$\underline{T}_{bB} = T_{bB}\, \hat{\underline{b}}_3 \qquad\qquad (9\text{-}11)$$

### 9.3.5 Control

$$\underline{T}_{cb} = -K_b (\theta_{xc} - \theta_x + \theta_F) \cos\theta_y\, \hat{\underline{b}}_3 \qquad\qquad (9\text{-}12)$$

Note that the control torque is responsive to the antenna error signal about the X axis with ship flexure again introduced as a worst-case effect. Any displacement in Y serves to increase the X rate response to a given torque input. This is offset by introducing $\cos\theta_y$ into Eq. (9-12).

### 9.3.6 Bearing Reaction

Following similar arguments to that in 9.2.6, we write

$$\underline{T}_{Rb} = T_{b1}\, \hat{\underline{b}}_1 \qquad\qquad (9\text{-}13)$$

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### 9.3.7  Total Torque on 'B' Gyro

The total torque on the 'B' gyro is

$$\underline{T}_b = T_b \, \hat{\underline{b}}_1 + T_{b3} \, \hat{\underline{b}}_3 \tag{9-14}$$

where

$$T_{b3} \, \hat{\underline{b}}_3 = \underline{T}_{bs} + \underline{T}_{bF} + \underline{T}_{bu} + \underline{T}_{bB} + \underline{T}_{cb}$$

### 9.4  Platform Torques

### 9.4.1  Spring

Spring torque acting on the platform is a function of $\theta_x$ and $\theta_y$ displacement.  Spring torque due to $\theta_T$ is cancelled out by the stepper motor.

$$\underline{T}_{ps} = -K_{sp} [\, \theta_x \, \hat{\underline{q}}_1 + \theta_y \, \hat{\underline{p}}_3 \,] \tag{9-15}$$

where $\quad K_{sp}$ = spring gradient

### 9.4.2  Friction

Friction is modelled as a constant amplitude torque with a polarity in opposition to velocities $\dot{\theta}_x$ and $\dot{\theta}_y$.

$$\underline{T}_{pF} = -K_{Fp} [\, \{\dot{\theta}_x / |\dot{\theta}_x|\} \, \hat{\underline{q}}_1 + \{\dot{\theta}_y / |\dot{\theta}_y|\} \, \hat{\underline{p}}_3 \,] \tag{9-16}$$

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### 9.4.3  Unbalance

The platform unbalance torque is defined in Section D-10 of Appendix D with the result

$$\underline{T}_{pu} = T_{u1} \, \hat{\underline{p}}_1 + T_{u2} \, \hat{\underline{p}}_2 + T_{u3} \, \hat{\underline{p}}_3 \qquad (9\text{-}17)$$

where

$$T_{u1} = M(A_2 \ell_3 - A_3 \ell_2)$$

$$T_{u2} = M(A_3 \ell_1 - A_1 \ell_3)$$

$$T_{u3} = M(A_1 \ell_2 - A_2 \ell_1)$$

$M$ = platform mass

$\ell_i$ = displacement of mass center along $\hat{\underline{p}}_i$ axis

### 9.4.4  Bias

$$\underline{T}_{pB} = T_{Bx} \, \hat{\underline{q}}_1 + T_{By} \, \hat{\underline{p}}_3 \qquad (9\text{-}18)$$

where  $T_{Bx}, T_{By}$ = bias torque about X and Y axes

### 9.4.5  Gyro Bearing Reaction

The total torques acting on the 'A' and 'B' gyros are defined as $\underline{T}_a$ and $\underline{T}_b$ in Eqs. (9-7) and (9-14) respectively. These are primarily reaction torques between the gyros and the platform except for the gyro unbalance torques which are small.

Accordingly, the gyro reaction torque acting on the platform is modelled by:

$$\underline{T}_{Rp} = -\underline{T}_a - \underline{T}_b \tag{9-19}$$

### 9.4.6 Control

Torquers on the X and Y axes are used to cage the gyro spin axes by nulling $\gamma_a$ and $\gamma_b$ using a proportional plus integral control loop.

$$\underline{T}_{pc} = -K_{pc}(\gamma_b \hat{\underline{q}}_1 + \gamma_a \hat{\underline{p}}_3) - K_{pI}(\int \gamma_b dt \, \hat{\underline{q}}_1 + \int \gamma_a dt \, \hat{\underline{p}}_3) \tag{9-20}$$

where   $K_{pc}$ = direct gain

   $K_{pI}$ = integrator gain

Note that:

$$\int \gamma_a dt = X_{10} \quad \text{and} \quad \int \gamma_b dt = X_{11}$$

### 9.4.7 Bearing Reaction

Radial forces at the X and Y axis bearings generate a torque about axis $\hat{\underline{q}}_2$. We define this term as

$$\underline{T}_{pG} = T_{pG} \, \hat{\underline{q}}_2 \tag{9-21}$$

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### 9.4.8 Total Platform Torque

Sum the previously defined torque contributions to determine the total platform torque:

$$\underline{T}_p = \underline{T}_{ps} + \underline{T}_{pF} + \underline{T}_{pu} + \underline{T}_{pB} + \underline{T}_{Rp} + \underline{T}_{pc} + \underline{T}_{pG} \tag{9-22}$$

or in terms of $\hat{\underline{p}}$ frame components,

$$\underline{T}_p = T_{p1}\,\hat{\underline{p}}_1 + T_{p2}\,\hat{\underline{p}}_2 + T_{p3}\,\hat{\underline{p}}_3 \tag{9-23}$$

## 10.0 SYSTEM EQUATIONS

The system equations are generated in the Appendices and are summarized below followed by a description of their development process.

$$\dot{X}_1 = \dot{\theta}_x = (\omega_{p1} - D_1\cos\theta_y - Y_2\sin\theta_y)/\cos\theta_y \tag{10-1}$$

where

$$D_1 = \omega_{f1}\cos\theta_T + \omega_{f2}\sin\theta_T$$

$$Y_2 = D_2\cos\theta_x + (\dot{\theta}_T + \omega_{f3})\sin\theta_x$$

$$D_2 = -\omega_{f1}\sin\theta_T + \omega_{f2}\cos\theta_T$$

$$\dot{X}_2 = \dot{\theta}_y = \omega_{p3} + D_2\sin\theta_x - (\dot{\theta}_T + \omega_{f3})\cos\theta_x \tag{10-2}$$

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$$\dot{X}_3 = \dot{\omega}_{a1} = J_{a1} + T_{a1}/I_{g1} \tag{10-3}$$

where $\quad J_{a1} = \omega_{a3}\Omega_a(I_{g2}/I_{g1}) - \omega_{a3}\omega_{a2}$

$$\dot{X}_4 = \dot{\omega}_{b3} = J_{b3} + T_{b3}/I_{g1} \tag{10-4}$$

where $\quad J_{b3} = \omega_{b1}\Omega_b(I_{g2}/I_{g1}) + \omega_{b1}\omega_{b2}$

$$\dot{X}_5 = \dot{\omega}_{p3} = \frac{d_{20}\dot{\omega}_{p2} + d_{22}}{1-d_{21}} \tag{10-5}$$

where $\quad d_i$ terms are defined in Appendix F

$$\dot{X}_6 = \dot{\omega}_{p1} = d_{11}/d_{12} \tag{10-6}$$

$$\dot{X}_7 = \dot{\omega}_{p2} = -\frac{d_{11}}{d_{12}}\tan\theta_y - d_{10}\sin\theta_y + d_{13} \tag{10-7}$$

$$\dot{X}_8 = \dot{\gamma}_A = \omega_{a1} - \omega_{p3} \tag{10-8}$$

$$\dot{X}_9 = \dot{\gamma}_b = \omega_{b3} - \omega_{p3} \tag{10-9}$$

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$$\dot{X}_{10} = \gamma_a \tag{10-10}$$

$$\dot{X}_{11} = \gamma_b \tag{10-11}$$

$$\dot{X}_{12} = \dot{\theta}_T \tag{10-12}$$

$$\dot{X}_{13} = \ddot{\theta}_T = -2\omega_n \zeta (\dot{\theta}_T + \dot{\psi}_A) + \omega_n^2 (\alpha_A - \theta_T - \psi_A) \tag{10-13}$$

The following table shows the source of development of the system equations:

| Equation(s) | Source |
|---|---|
| 10-1, 10-2 | Appendix E |
| 10-3, 10-4 | Sect. 5.0 & Appendix A |
| 10-5, 10-6, 10-7 | Appendix F |
| 10-8, 10-9 | Section 3.0 |
| 10-10, 10-11 | Equation (9-20) |
| 10-12, 10-13 | Equation (8-12) |

## 11.0 COMPUTER SIMULATION

The computer program based on the preceding analysis is described and listed in Appendix H.

### 11.1 Parametric Values

Parameter values are shown below for two cases. The variable designation in front of each corresponds to that used in the computer simulation.

Case 1:  Ship rolling and flexing

Case 2:  Ship rolling, pitching and turning at high speed. No flexing included.

|  |  |  | Case 1 | Case 2 |
|---|---|---|---|---|
| D(1) | = | Desired Azim (Deg) | 0 | 0 |
| D(2) | = | Desired Elev (Deg) | 57.3 | 57.3 |
| D(3) | = | Roll Amplit (Deg) | 20.0 | 20.0 |
| D(4) | = | Pitch Amplit (Deg) | 0 | 10.0 |
| D(5) | = | Flex Amplit (Deg) | .5 | 0 |
| D(6) | = | Roll Period (Sec) | 10.0 | 10.0 |
| D(7) | = | Pitch Period (Sec) | - | 15.0 |
| D(8) | = | Flex Period (Sec) | 1.0 | - |
| D(9) | = | Roll Height (In) | 1200.0 | 1200.0 |
| D(10) | = | Velocity (Knots) | 0 | 20.0 |
| D(11) | = | Turn Rate (Deg/Sec) | 0 | 3.0 |
| D(12) | = | Heel Angle (Deg) | 0 | 0 |
| D(13) | = | Cant Angle (Deg) | 0 | 0 |

|          |   |                                  |      |      |
|----------|---|----------------------------------|------|------|
| D(14)    | = | WN Train Servo (Rad/Sec)         | 3.   | 3.   |
| D(15)    | = | Damping Ratio Train Servo        | .7   | .7   |

The values below were used in both cases:

|        |   |                                           |        |
|--------|---|-------------------------------------------|--------|
| E(1)   | = | Platform Mass (Lb-Sec2/In)                | 0.25   |
| E(2)   | = | Gyro Mass (Lb-Sec$^2$/In)                 | 0.06   |
| E(3)   | = | Gyro Mass Offset (In)                     | 0.003  |
| E(4)   | = | Plat 1 Mass Offset (In)                   | 0.003  |
| E(5)   | = | Plat 2 Mass Offset (In)                   | 0.003  |
| E(6)   | = | Plat 3 Mass Offset (In)                   | 0.003  |
| E(7)   | = | Gyro Friction (In-Lb)                     | 0.04   |
| E(8)   | = | Gyro Spring (In-Lb/Rad)                   | 0.20   |
| E(9)   | = | Gyro A Control Gain (In-Lb/Rad)           | 108.0  |
| E(10)  | = | Gyro B Control Gain (In-Lb/Rad)           | 108.0  |
| E(11)  | = | Platform Friction (In-Lb)                 | 0.5    |
| E(12)  | = | Platform Spring (In-Lb/Rad)               | 1.5    |
| E(13)  | = | Gyro Inertia 1 and 3 (Lb-In-Sec2)         | 0.5    |
| E(14)  | = | Gyro Inertia 2                            | 1.0    |
| E(15)  | = | Platform Inertia 1                        | 40.0   |
| E(16)  | = | Platform Inertia 2                        | 30.0   |
| E(17)  | = | Platform Inertia 3                        | 40.0   |
| E(18)  | = | Gyro Spin Speed (Rad/Sec)                 | 361.0  |

| | | | |
|---|---|---|---|
| E(19) | = | Platf Contr Gain (In-Lb/Rad) | 50.0 |
| E(20) | = | Platf Torq Integr Gain (In-Lb/Rad-Sec) | 5.0 |
| E(21) | = | Gyro A Torque Bias (In-Lb) | 0 |
| E(22) | = | Gyro B Torque Bias (In-Lb) | 0 |
| E(23) | = | Plat Elev Bias Torque (In-Lb) | 0 |
| E(24) | = | Plat Cross Elev Bias (In-Lb) | 0 |
| E(25) | = | Limit on Gyro Torquer (In-Lb) | 5.0 |
| E(26) | = | Gyro Speed Mismatch | 0 |

## 11.2  Initial State Values

In both cases the system is initialized with the antenna pointing at the satellite $(X_1(0) = \theta_x(0) = 1.$ radian; $D(2) = \alpha_A = 57.3^O)$ and with a body rate, $\omega_{p2}$, necessary to match the $\hat{p}_2$ component of the ship's roll rate.

### Results

### Case 1

Figure 6 shows a plot of the X and Y axis error over a period of 75 seconds.  The error is the difference between the computed command angle and the actual angle determined by the integration process.  Figure 7 shows the motion of the 'A' and 'B' gyros about their pivot axes during the same period.  There are two aspects of the response to be examined.

The ±0.5 degree ship flexure input is attenuated to approximately ±0.02 degree  of X-Y error by the narrow bandwidth transfer function of the overall system.

The roll input with a 10-second period causes oscillations at that period in the X-Y error and the 'A' gyro motion.  The 'B' gyro shows a double frequency response with a 5-second period.

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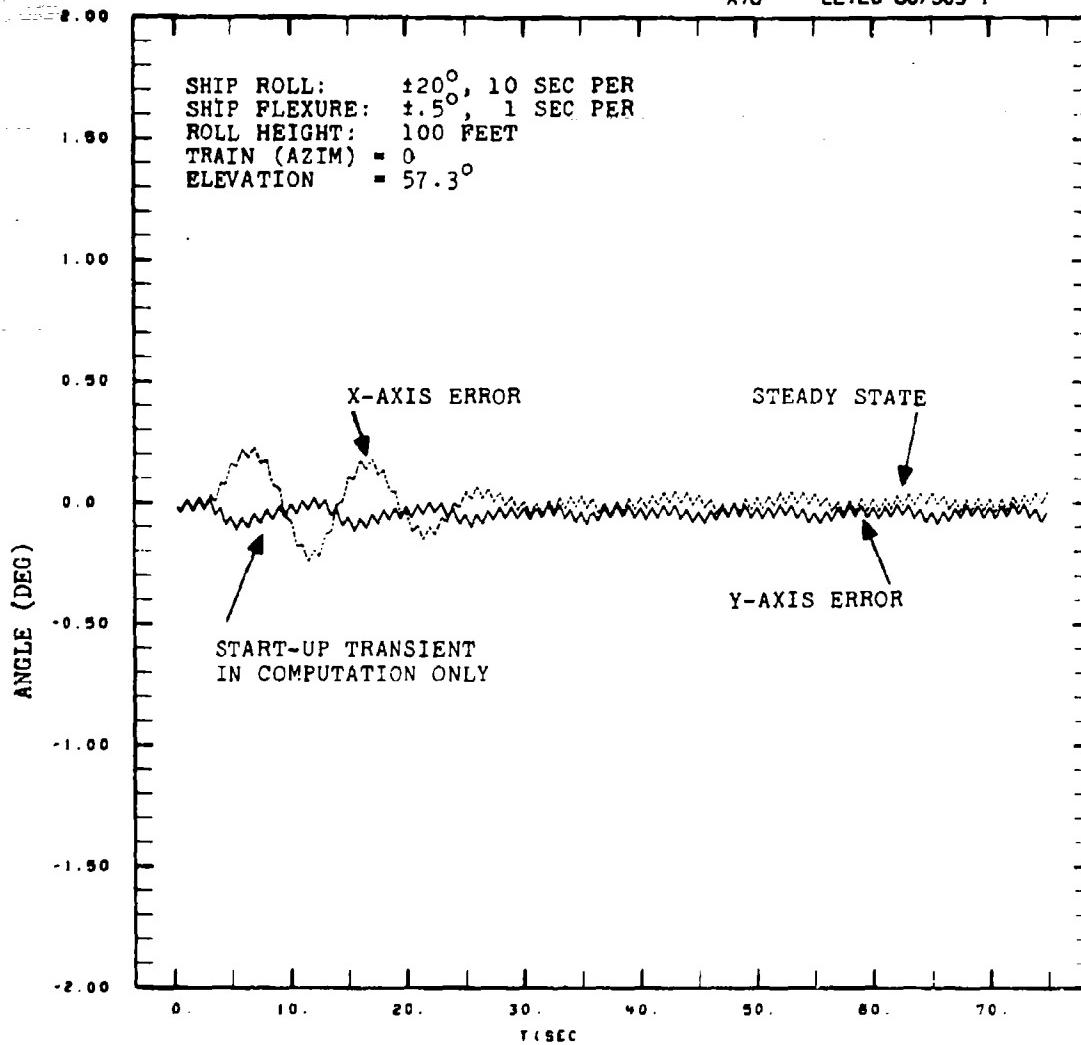
Fig. 6.   X-Y DMS error (rolling).

X76    22:20 80/309 2
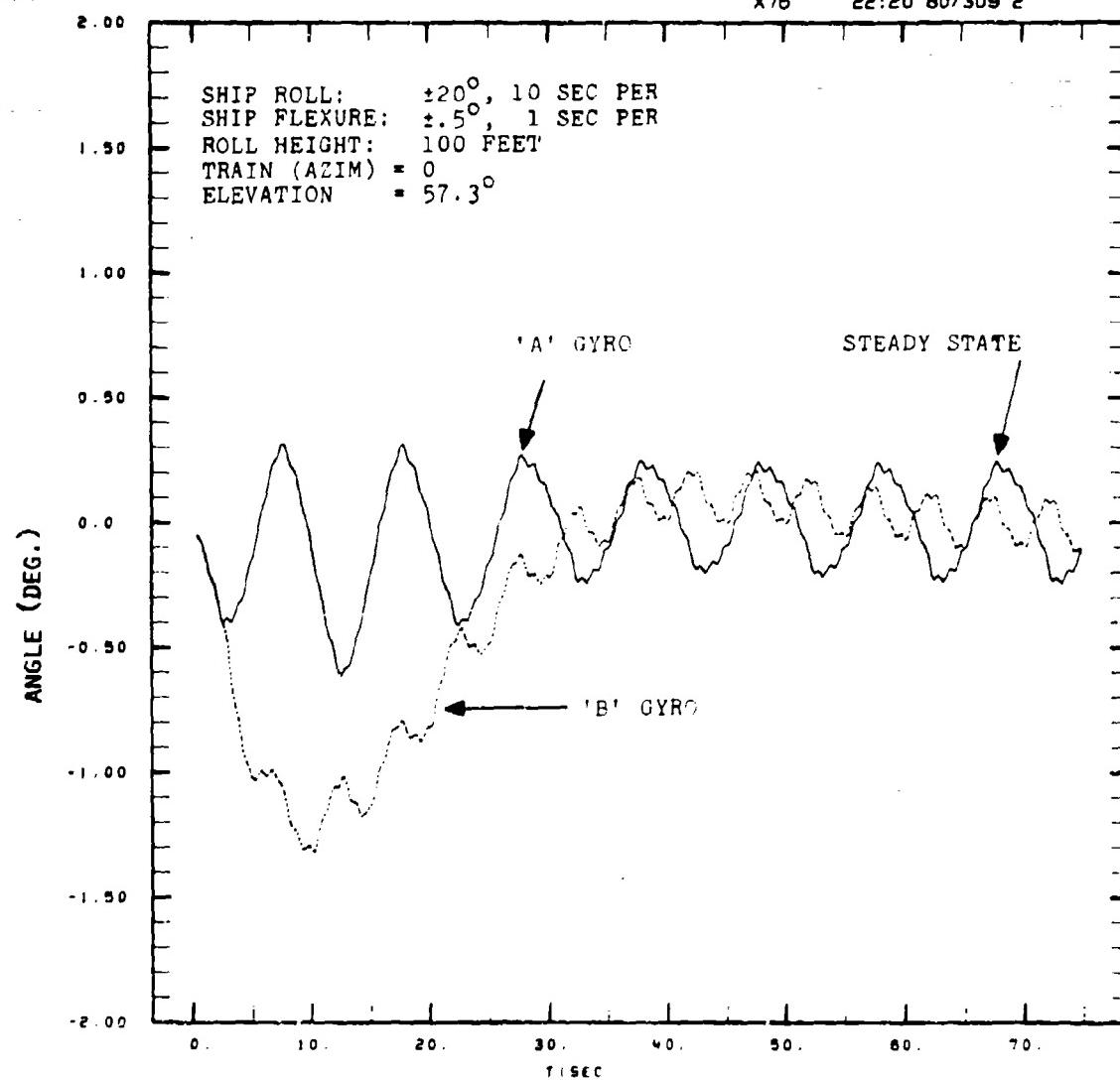


Fig. 7.   X-Y DMS gyro motion (rolling).

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After an initial transient a steady state response is achieved
in approximately 30 seconds with an X-Y error averaging less
than 0.05°.

## Case 2

Figures 8 and 9 show the X-Y errors and the gyro motion over
a 90-second period.  This is a more confused dynamic environment
than in Case 1 and, accordingly, we see a more confused response
in Figures 8 and 9.

After an initial transient a general steady state band of
response is achieved in approximately 60 seconds with an average
X-Y error of less than 0.1 degree.
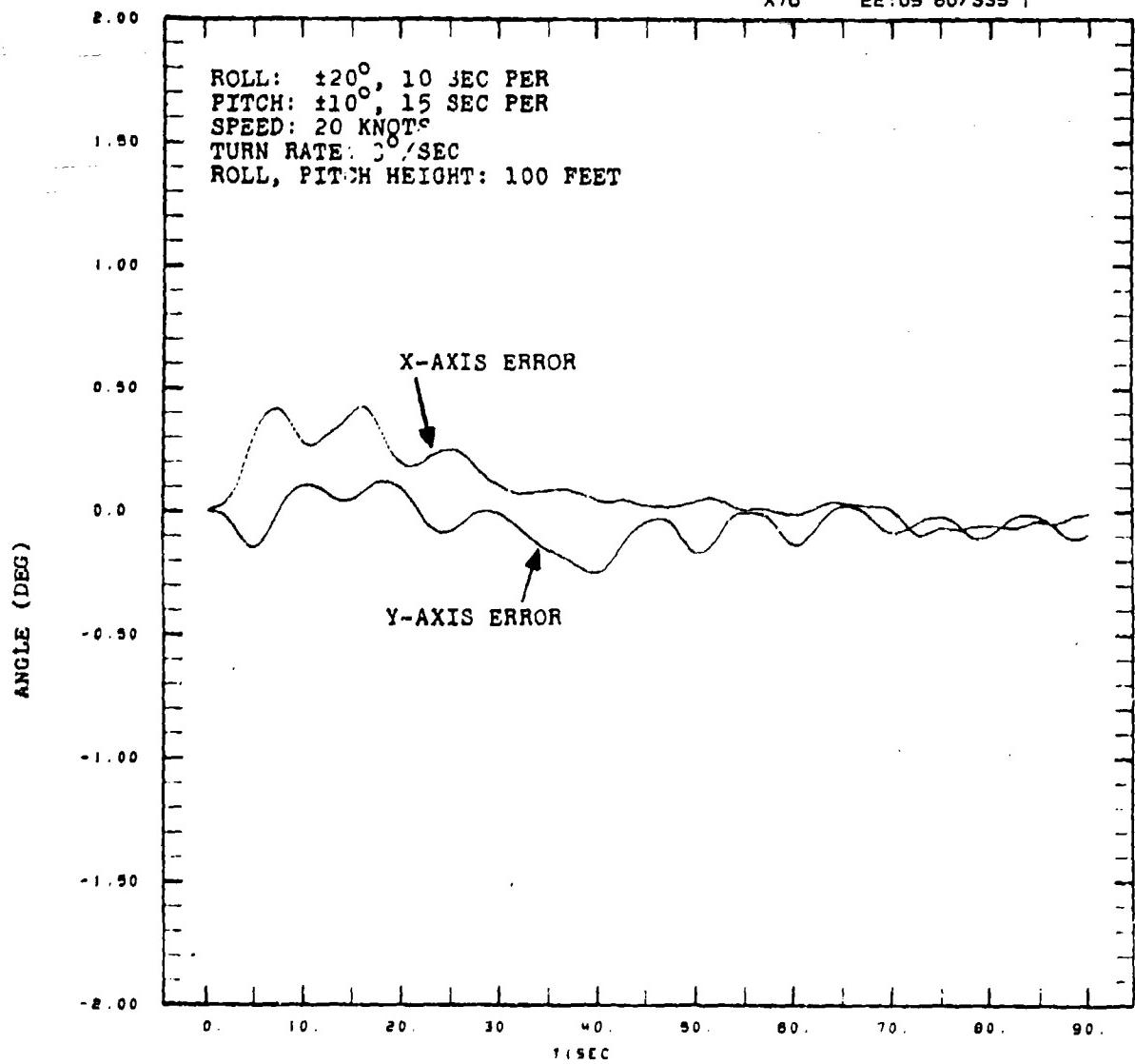
Fig. 8.   X-Y DMS error (roll, pitch, turn).
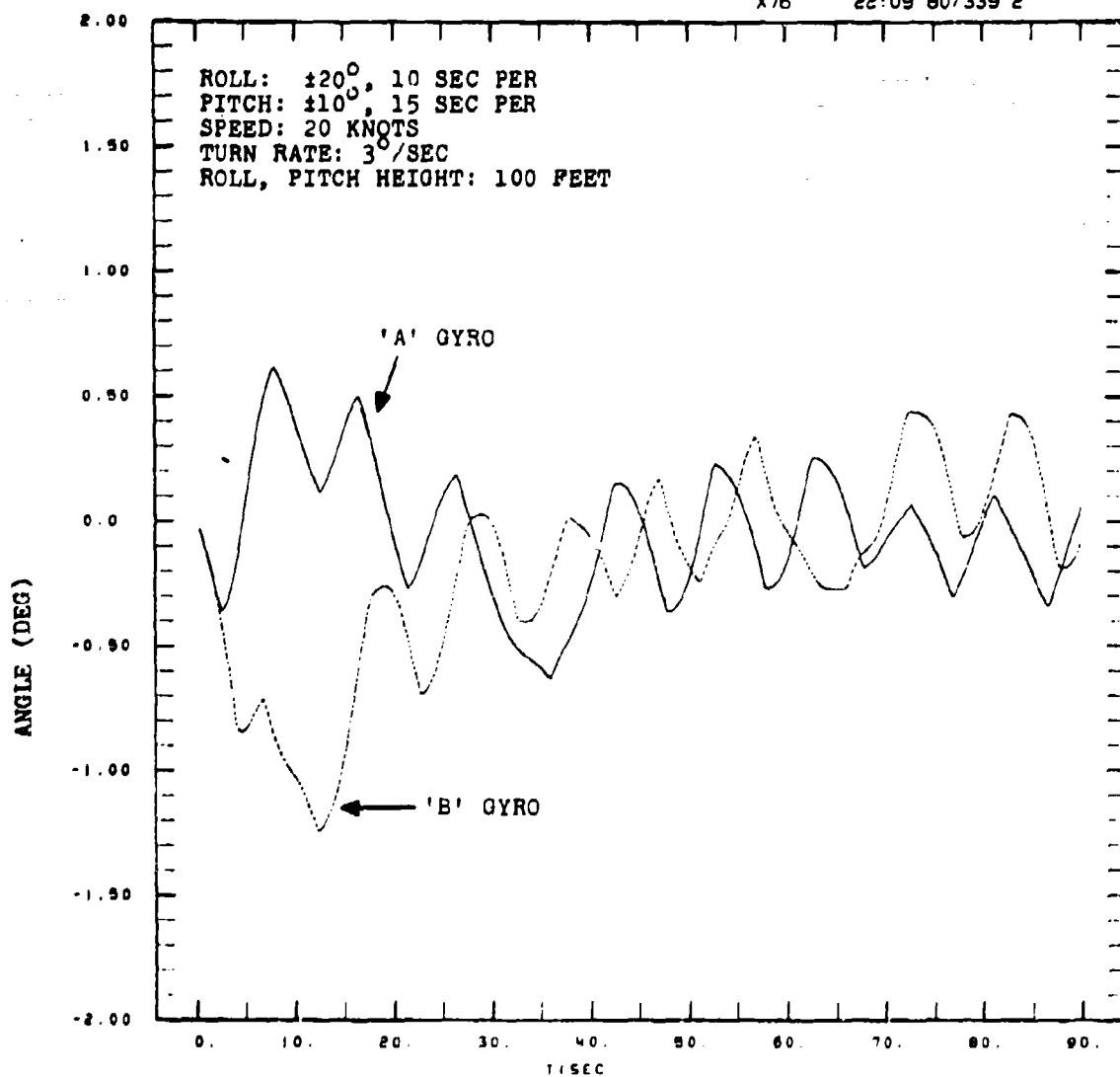
Fig. 9.   X-Y DMS gyro motion (roll, pitch, turn).

# APPENDIX A

## EULER EQUATIONS FOR GYROS AND PLATFORM

### 1.  Gyro 'A'

The Euler equations[*] for gyro 'A' are written in tensor form
as

$$\underline{\hat{a}} \frac{d}{dt} (\underline{\Omega}_a) = -\underline{I}_g^{-1} [\tilde{\omega}_a] \underline{I}_g \underline{\Omega}_a + \underline{I}_g^{-1} \underline{T}_a \tag{A-1}$$

where $\quad \underline{\Omega}_a = \omega_{a1} \underline{\hat{a}}_1 + \Omega_a \underline{\hat{a}}_2 + \omega_{a3} \underline{\hat{a}}_3 \tag{3-6}$

$\Omega_a$ = constant rotor spin speed

$$\underline{I}_g = \begin{bmatrix} I_{g1} & 0 & 0 \\ 0 & I_{g2} & 0 \\ 0 & 0 & I_{g1} \end{bmatrix}$$

$$[\tilde{\omega}_a] = \begin{bmatrix} 0 & -\omega_{a3} & \omega_{a2} \\ \omega_{a3} & 0 & -\omega_{a1} \\ -\omega_{a2} & \omega_{a1} & 0 \end{bmatrix}$$

$$\underline{T}_a = T_{a1} \underline{\hat{a}}_1 + T_{a2} \underline{\hat{a}}_2 + T_{a3} \underline{\hat{a}}_3 \tag{9-7}$$

Differentiating Eq. (3-6) gives

_____

[*]  See Ref. (7)

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$$\frac{d}{dt} (\underline{\Omega}_a) = \dot{\omega}_{a1} \hat{\underline{a}}_1 + \dot{\omega}_{a3} \hat{\underline{a}}_3 \qquad (A-2)$$

Combine (A-1), (A-2) and the terms defined above to get

$$
\begin{Bmatrix} \dot{\omega}_{a1} \\ 0 \\ \dot{\omega}_{a3} \end{Bmatrix} = -
\begin{bmatrix} 1/I_{g1} & 0 & \\ 0 & 1/I_{g2} & 0 \\ 0 & 0 & 1/I_{g1} \end{bmatrix}
\begin{bmatrix} 0 & -\omega_{a3} & \omega_{a2} \\ \omega_{a3} & 0 & -\omega_{a1} \\ -\omega_{a2} & \omega_{a1} & 0 \end{bmatrix}
$$

$$
\begin{bmatrix} I_{g1} & 0 & 0 \\ 0 & I_{g2} & 0 \\ 0 & 0 & I_{g1} \end{bmatrix}
\begin{Bmatrix} \omega_{a1} \\ \Omega_a \\ \omega_{a3} \end{Bmatrix} +
$$

$$
\begin{bmatrix} 1/I_{g1} & 0 & 0 \\ 0 & 1/I_{g2} & 0 \\ 0 & 0 & 1/I_{g1} \end{bmatrix}
\begin{Bmatrix} T_{a1} \\ T_{a2} \\ T_{a3} \end{Bmatrix} \qquad (A-3)
$$

After some algebra the second of the (A-3) equations vanishes leaving:

$$\dot{\omega}_{a1} = 1/I_{g1} [\omega_{a3}\Omega_a I_{g2} - \omega_{a2}\omega_{a3}I_{g1} + T_{a1}] \qquad (A-4)$$

$$\dot{\omega}_{a3} = 1/I_{g1} [\omega_{a2}\omega_{a1}I_{g1} - \omega_{a1}\Omega_a I_{g2} + T_{a3}] \qquad (A-5)$$

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which can then be put into the form of Eqs. (5-1) and (5-2)

$$\dot{\omega}_{a1} = J_{a1} + T_{a1}/I_{g1} \tag{5-1}$$

$$\dot{\omega}_{a3} = J_{a3} + T_{a3}/I_{g3} \tag{5-2}$$

where

$$J_{a1} = \omega_{a3}(\Omega_a I_{g2} - \omega_{a2} I_{g1})/I_{g1}$$

$$J_{a3} = \omega_{a1}(-\Omega_a I_{g2} + \omega_{a2} I_{g1})/I_{g1}$$

2. <u>Gyro 'B'</u>

The development of Eqs. (5-3) and (5-4) for the 'B' gyro is similar starting with

$$\overset{\wedge}{\underline{b}} \frac{d}{dt} (\underline{\Omega}_b) = -\underline{I}_g^{-1} [\tilde{\omega}_b] \underline{I}_g \underline{\Omega}_b + \underline{I}_g^{-1} \underline{T}_b \tag{A-6}$$

3. <u>Platform</u>

The Euler equations for the platform are

$$\dot{\underline{\omega}}_p = -\underline{I}_p^{-1} [\tilde{\omega}_p] \underline{I}_p \underline{\omega}_p + \underline{I}_g^{-1} \underline{T}_p \tag{A-7}$$

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where $\quad \underline{\omega}_p = \omega_{p1} \hat{\underline{p}}_1 + \omega_{p2} \hat{\underline{p}}_2 + \omega_{p3} \hat{\underline{p}}_3 \qquad (3-3)$

$$\underline{I}_p = \begin{bmatrix} I_{p1} & 0 & 0 \\ 0 & I_{p2} & 0 \\ 0 & 0 & I_{p3} \end{bmatrix} \quad \text{in } \hat{\underline{p}} \text{ frame}$$

$$\tilde{\omega}_p = \begin{bmatrix} 0 & -\omega_{p3} & \omega_{p2} \\ \omega_{p3} & 0 & -\omega_{p1} \\ -\omega_{p2} & \omega_{p1} & 0 \end{bmatrix}$$

$$\underline{T}_p = T_{p1} \hat{\underline{p}}_1 + T_{p2} \hat{\underline{p}}_2 + T_{p3} \hat{\underline{p}}_3 \qquad (9-23)$$

There are no gyroscopic terms in (A-7) since the platform motions are relatively slow, therefore expansion of (A-7) leads to

$$\dot{\underline{\omega}}_p = \dot{\omega}_{p1} \hat{\underline{p}}_1 + \dot{\omega}_{p2} \hat{\underline{p}}_2 + \dot{\omega}_{p3} \hat{\underline{p}}_3 \qquad (A-8)$$

where
$$\left. \begin{array}{l} \dot{\omega}_{p1} = T_{p1}/I_{p1} \\[2mm] \dot{\omega}_{p2} = T_{p2}/I_{p2} \\[2mm] \dot{\omega}_{p3} = T_{p3}/I_{p3} \end{array} \right\} \qquad (A-9)$$

Equations (A-9) are Eqs. (6-1), (6-2), (6-3).

## APPENDIX B

## CONSTRAINTS

The angular velocities of the $\{\hat{\underline{a}}\}$ and $\{\hat{\underline{b}}\}$ frames are constrained to that of the $\{\hat{\underline{p}}\}$ frame by the gimbal arrangement shown in Figure 3.

Consider the $\{\hat{\underline{a}}\}$ frame which pivots about $\hat{\underline{a}}_1$. The component $\omega_{a1}$ is not constrained; however, $\omega_{a2}$ and $\omega_{a3}$ are by the equality of the vector sums:

$$\omega_{a2}\,\hat{\underline{a}}_2 + \omega_{a3}\,\hat{\underline{a}}_3 = \omega_{p2}\,\hat{\underline{p}}_2 + \omega_{p3}\,\hat{\underline{p}}_3 \tag{B-1}$$

Using the relations defined in Section 2.0 we can write

$$\{\hat{\underline{a}}\} = [C_1(\gamma_a)]\,\{\hat{\underline{p}}\}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma_a & \sin\gamma_a \\ 0 & -\sin\gamma_a & \cos\gamma_a \end{bmatrix} \begin{Bmatrix} 0 \\ \omega_{p2} \\ \omega_{p3} \end{Bmatrix}_{\hat{\underline{p}}} = \begin{Bmatrix} 0 \\ \omega_{p2}\cos\gamma_a + \omega_{p3}\sin\gamma_a \\ -\omega_{p2}\sin\gamma_a + \omega_{p2}\cos\gamma_a \end{Bmatrix}_{\hat{\underline{a}}} \tag{B-2}$$

Combining (B-1) and (B-2) we get the equations

$$\omega_{a2} = \omega_{p2}\cos\gamma_a + \omega_{p3}\sin\gamma_a \qquad (B-3)$$

$$\omega_{a3} = -\omega_{p2}\sin\gamma_a + \omega_{p3}\cos\gamma_a \qquad (B-4)$$

Equations (B-3) and (B-4) are Eqs. (7-1) and (7-2). Similarly for frame $\{\hat{\underline{b}}\}$ which has a single degree of freedom with respect to $\{\hat{\underline{p}}\}$ about axis $\underline{b}_3$ we get

$$\omega_{b1}\,\hat{\underline{b}}_1 + \omega_{b2}\,\hat{\underline{b}}_2 = \omega_{p1}\,\hat{\underline{p}}_1 + \omega_{p2}\,\hat{\underline{p}}_2 \qquad (B-5)$$

$$\{\hat{\underline{b}}\} = [C_3(\gamma_b)]\,\{\hat{\underline{p}}\}$$

$$\begin{bmatrix} \cos\gamma_b & \sin\gamma_b & 0 \\ -\sin\gamma_b & \cos\gamma_b & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \omega_{p1} \\ \omega_{p2} \\ 0 \end{Bmatrix}_{\hat{\underline{p}}} = \begin{Bmatrix} \omega_{p1}\cos\gamma_b + \omega_{p2}\sin\gamma_b \\ -\omega_{p1}\sin\gamma_b + \omega_{p2}\cos\gamma_b \\ 0 \end{Bmatrix}_{\hat{\underline{b}}} \qquad (B-6)$$

$$\omega_{b1} = \omega_{p1}\cos\gamma_b + \omega_{p2}\sin\gamma_b \qquad (B-7)$$

$$\omega_{b2} = -\omega_{p1}\sin\gamma_b + \omega_{p2}\cos\gamma_b \qquad (B-8)$$

Equations (B-7) and (B-8) are Eqs. (7-3) and (7-4).

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# APPENDIX C

## TRANSFORMATIONS FOR $\theta_{xc}$ AND $\theta_{yc}$

In this appendix we expand Eq. (8-8) and solve the results for $\theta_{xc}$ and $\theta_{yc}$.

$$B_1^T B_2^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} = B_3^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} \qquad (8-8)$$

The unknowns $\theta_{xc}$ and $\theta_{yc}$ are contained in $B_2$ and so we premultiply by $B_1$ to get

$$B_2^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} = B_1 B_3^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} \qquad (C-1)$$

Expand the left side

$$B_2^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} = [C_1(\theta_{xc})]^T [C_3(\theta_{yc})]^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$

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$$B_2^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{xc} & -\sin\theta_{xc} \\ 0 & \sin\theta_{xc} & \cos\theta_{xc} \end{bmatrix} \begin{bmatrix} \cos\theta_{yc} & -\sin\theta_{yc} & 0 \\ \sin\theta_{yc} & \cos\theta_{yc} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$

$$= \begin{bmatrix} -\sin\theta_{yc} \\ \cos\theta_{xc} \cos\theta_{yc} \\ \sin\theta_{xc} \cos\theta_{yc} \end{bmatrix} \tag{C-2}$$

Expand the right side of (C-1)

$$B_1 B_3^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} = [C_3(\theta_T)][C_2(\psi_{RH})][C_1(\psi_P)][C_3(\psi_A)][C_1(\alpha_A)]^T [C_1(\alpha_E)]^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} t_1 \\ t_2 \\ t_3 \end{Bmatrix} \tag{C-3}$$

The $t_i$ terms will be determined by numerical computation of (C-3). Then equate the first and third components of (C-2) and (C-3).

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$$t_1 = -\sin\theta_{yc}$$

$$t_3 = \sin\theta_{xc}\cos\theta_{yc}$$

Solve for $\theta_{yc}$ and $\theta_{xc}$

$$\left.\begin{aligned} \theta_{yc} &= \sin^{-1}(-t_i) \\[2em] \theta_{xc} &= \sin^{-1}(t_3/\cos\theta_{yc}) \end{aligned}\right\} \qquad\qquad (C-4)$$

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# APPENDIX D

## ACCELERATION VECTOR & TORQUE DUE TO UNBALANCE

We consider accelerations due to the following sources:

- a) Roll     (horizontal component along $\hat{\underline{n}}_1'$)
- b) Pitch    (horizontal component along $\hat{\underline{n}}_2'$)
- c) Gravity (vertical along $\hat{\underline{n}}_3'$)
- d) Turn     (horizontal component along $\hat{\underline{n}}_1'$)

1.  Roll Acceleration (horizontal component along $\hat{\underline{n}}_1'$)

$$\underline{A}_R = H\ \ddot{\psi}_R\ \hat{\underline{n}}_1' \tag{D-1}$$

where $\quad \ddot{\psi}_R = -\omega_R^2 \psi_{RO} \sin\omega_R t$

$\qquad H = $ roll height

$\qquad \omega_R = $ roll frequency

$\qquad \psi_{RO} = $ roll amplitude

2.  Pitch Acceleration (horizontal component along $\underline{n}_2'$)

$$\underline{A}_p = H\ \ddot{\psi}_p\ \hat{\underline{n}}_2' \tag{D-2}$$

where $\quad \ddot{\psi}_p = -\omega_p^2 \psi_{p0} \sin \omega_p t$

$\omega_p$ = pitch frequency

$\psi_{p0}$ = pitch amplitude

3.  Gravity (along $\hat{\underline{n}}_3'$)

$$\underline{A}_g = 386 \text{ in/sec}^2 \cdot \hat{\underline{n}}_3' \qquad \text{(D-3)}$$

Note on Polarity of $\underline{A}_g$:

Force due to gravity acts down at the center of mass. Its corresponding support force at the pivot axis is positive in the upward direction. Torque about the mass center results when the line of the positive support force does not pass through the mass center.

4.  Turn (horizontal component along $\underline{n}_1'$)

$$\underline{A}_T = -V \omega_T \hat{\underline{n}}_1' \qquad \text{(D-4)}$$

where

$V$ = velocity

$\omega_T$ = turn rate

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5.   Total Acceleration in $\{\hat{\underline{n}}'\}$

We add components above to obtain

$$\underline{A} = A_1' \; \hat{\underline{n}}_1' + A_2' \; \hat{\underline{n}}_2' + A_3' \; \hat{\underline{n}}_3' \qquad\qquad\qquad (D\text{-}5)$$

where     $A_1' = H \; \ddot{\psi}_R - V \; \omega_T$

$A_2' = H \; \ddot{\psi}_p$

$A_3' = 386$

6.   Transform Acceleration Vector to $\{\hat{\underline{p}}\}$

We transform from $\{\hat{\underline{n}}'\}$ to $\{\hat{\underline{p}}\}$ using

$$\{\hat{\underline{p}}\} = [C_3(\theta_y)][C_1(\theta_x)][C_3(\theta_T)][C_2(\psi_R)][C_1(\psi_p)] \; \{\hat{\underline{n}}'\} \qquad (D\text{-}6)$$

and define the resulting components in $\{\hat{\underline{p}}\}$ by

$$\underline{A} = A_1 \; \hat{\underline{p}}_1 + A_2 \; \hat{\underline{p}}_2 + A_3 \; \hat{\underline{p}}_3 \qquad\qquad\qquad (D\text{-}7)$$

7.   Unbalance Torque

Unbalance torque due to the combined effect of acceleration and displacement berween the centers of support and mass is defined by the cross product:

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$$\underline{L} = \underline{F} \times \underline{R} \tag{D-8}$$

where   $\underline{L}$ = unbalance torque vector

$\underline{F}$ = force vector at support point

$\underline{R}$ = position vector from support point to mass center

The force vector, $\underline{F}$, is due to the effect of gravity and acceleration acting on the mass.

$$\underline{F} = m \, \underline{A} \tag{D-9}$$

8.   'A' Gyro Unbalance Torque

Transform $\underline{A}$ in Eq. (D-7) from $\{\hat{\underline{p}}\}$ to $\{\hat{\underline{a}}\}$ coordinates using

$$\{\hat{\underline{a}}\} = [C_1(\gamma_a)] \{\hat{\underline{p}}\} \tag{D-10}$$

$$\{\underline{A}\}_{\hat{\underline{a}}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma_a & \sin\gamma_a \\ 0 & -\sin\gamma_a & \cos\gamma_a \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}_{\hat{\underline{p}}} = \begin{Bmatrix} A_1 \\ A_2\cos\gamma_a + A_3\sin\gamma_a \\ -A_2\sin\gamma_a + A_3\cos\gamma_a \end{Bmatrix}_{\hat{\underline{a}}} \tag{D-11}$$

Write the result of (D-11) as

$$\underline{A} = A_1 \, \hat{\underline{a}}_1 + a_2 \, \hat{\underline{a}}_2 + a_3 \, \hat{\underline{a}}_3 \tag{D-12}$$

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where $\qquad a_2 = A_2 \cos\gamma_a + A_3 \sin\gamma_a$

$$a_3 = -A_2 \sin\gamma_a + A_3 \cos\gamma_a$$

Now consider

$$\underline{R} = \ell_g\, \underline{\hat{a}}_2$$

$$\underline{F} = m\, \underline{A} = m(A_1\, \underline{\hat{a}}_1 + a_2\, \underline{\hat{a}}_2 + a_3\, \underline{\hat{a}}_3)$$

where $\qquad \ell_g$ = displacement of mass center from support point

$\qquad m$ = gyro mass

then $\qquad \underline{T}_{au} = \underline{F} \times \underline{R} = m\ell_g(A_1\, \underline{\hat{a}}_3 - a_3\, \underline{\hat{a}}_1)$  (D-13)

The $\underline{\hat{a}}_3$ component in (D-13) is an insignificant bearing reaction torque which can be neglected. Therefore using the definition of $a_3$ in (D-12) we can write

$$\underline{T}_{au} = -m\ell_g(-A_2 \sin\gamma_a + A_3 \cos\gamma_a) \tag{D-14}$$

9. 'B' Gyro Unbalance Torque

Transform $\underline{A}$ in Eq. (D-7) from $\{\hat{\underline{p}}\}$ to $\{\hat{\underline{b}}\}$ using

$$\{\hat{\underline{b}}\} = [C_3(\gamma_b)]\, \{\hat{\underline{p}}\} \tag{D-15}$$

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$$\{\underline{A}\}_{\hat{\underline{b}}} = \begin{bmatrix} \cos\gamma_b & \sin\gamma_b & 0 \\ -\sin\gamma_b & \cos\gamma_b & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \end{Bmatrix}_{\hat{\underline{p}}} = \begin{Bmatrix} A_a\cos\gamma_b + A_2\sin\gamma_b \\ -A_1\sin\gamma_b - A_2\cos\gamma_b \\ A_3 \end{Bmatrix}_{\hat{\underline{b}}} \quad \text{(D-16)}$$

Write the result of (D-16) in $\{\hat{\underline{b}}\}$ coordinates as

$$\underline{A} = b_1 \hat{\underline{b}}_1 + b_2 \hat{\underline{b}}_2 + A_3 \hat{\underline{b}}_3 \quad \text{(D-17)}$$

Now consider

$$\underline{R} = \ell_g \hat{\underline{b}}_2$$

$$\underline{F} = m \underline{A} = m(b_1 \hat{\underline{b}}_1 + b_2 \hat{\underline{b}}_2 + A_3 \hat{\underline{b}}_3)$$

Then $\qquad \underline{T}_{bu} = \underline{F} \times \underline{R} = m\ell_g(-b_1 \hat{\underline{b}}_3 + A_3 \hat{\underline{b}}_1) \qquad \text{(D-18)}$

We neglect the $\hat{\underline{b}}_1$ component following the same logic that preceded Eq. (D-14). Then use the definition of $b_1$ in Eq. (D-17) to obtain

$$\underline{T}_{bu} = -m\ell_g(A_1\cos\gamma_b + A_2\sin\gamma_b) \quad \text{(D-19)}$$

10. Platform Unbalance Torque

Here we model unbalance about all three axes by defining

$$\underline{R} = \ell_1 \hat{\underline{P}}_1 + \ell_2 \hat{\underline{P}}_2 + \ell_3 \hat{\underline{P}}_3$$

now

$$\underline{T}_{pu} = \underline{F} \times \underline{R} = M(A_1\hat{\underline{P}}_1 + A_2\hat{\underline{P}}_2 + A_3\hat{\underline{P}}_3) \times (\ell_1\hat{\underline{P}}_1 + \ell_2\hat{\underline{P}}_2 + \ell_3\hat{\underline{P}}_3)$$

$$\underline{T}_{pu} = T_{u1} \hat{\underline{P}}_1 + T_{u2} \hat{\underline{P}}_2 + T_{u3} \hat{\underline{P}}_3 \qquad \text{(D-20)}$$

where

$$T_{u1} = M(A_2\ell_3 - A_3\ell_2)$$

$$T_{u2} = M(A_3\ell_1 - A_1\ell_3)$$

$$T_{u3} = M(A_1\ell_2 - A_2\ell_1)$$

$$M = \text{platform mass}$$

# APPENDIX E

## GENERATION OF SYSTEM EQUATIONS (10-1) AND (10-2) FOR $\dot{\theta}_x$ AND $\dot{\theta}_y$

### 1. Approach

System equations (10-1) and (10-2) are integrated to provide $\theta_x$ and $\theta_y$ position data. In the real system $\theta_x$ and $\theta_y$ would be the output of the synchros on those axes.

The solution for $\dot{\theta}_x$ and $\dot{\theta}_y$ is obtained by using two of the three scalar equations in the vector equation (3-4)

$$\underline{\omega}_p = \underline{\omega}_f + \dot{\theta}_T \, \hat{\underline{f}}'_3 + \dot{\theta}_x \, \hat{\underline{q}}_1 + \dot{\theta}_y \, \hat{\underline{p}}_3 \tag{3-4}$$

The process is one of defining $\underline{\omega}_f$, putting all components of (3-4) into a common frame $\{\hat{\underline{p}}\}$ and solving for $\dot{\theta}_x$ and $\dot{\theta}_y$ as functions of $\omega_{pi}$, $\omega_{fi}$, $\dot{\theta}_T$, $\theta_x$ and $\theta_y$.

### 2. Generate $\omega_{fi}$ Components

First generate the $\omega_{fi}$ components in Eq. (3-1)

$$\underline{\omega}_f = \omega_{f1} \, \hat{\underline{f}}_1 + \omega_{f2} \, \hat{\underline{f}}_2 + \omega_{f3} \, \hat{\underline{f}}_3 \tag{3-1}$$

by making use of Eq. (3-2)

$$\underline{\omega}_f = \dot{\psi}_A \, \hat{\underline{n}}'_3 + \dot{\psi}_p \, \hat{\underline{n}}''_1 + \dot{\psi}_R \, \hat{\underline{f}}_2 \tag{3-2}$$

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Transform $\dot{\psi}_A \, \hat{\underline{n}}'_3$ into $\{\hat{\underline{n}}''\}$ using from Section 2.0

$$\{\hat{\underline{n}}''\} = [C_1(\psi_p)] \, \{\hat{\underline{n}}'\}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi_p & \sin\psi_p \\ 0 & -\sin\psi_p & \cos\psi_p \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \dot{\psi}_A \end{Bmatrix}_{\hat{\underline{n}}'} = \begin{Bmatrix} 0 \\ \dot{\psi}_A \sin\psi_p \\ \dot{\psi}_A \cos\psi_p \end{Bmatrix}_{\hat{\underline{n}}''} \qquad (E-1)$$

Next add $\dot{\psi}_p \, \hat{\underline{n}}''_1$ to (E-1) and transform to $\{\hat{\underline{f}}\}$ using from Section 2.0

$$\{\hat{\underline{f}}\} = [C_2(\psi_R)] \, \{\hat{\underline{n}}''\}$$

$$\begin{bmatrix} \cos\psi_R & 0 & -\sin\psi_R \\ 0 & 1 & 0 \\ \sin\psi_R & 0 & \cos\psi_R \end{bmatrix} \begin{Bmatrix} \dot{\psi}_p \\ \dot{\psi}_A \sin\psi_p \\ \dot{\psi}_A \cos\psi_p \end{Bmatrix}_{\hat{\underline{n}}''} = \begin{Bmatrix} \dot{\psi}_p \cos\psi_R - \dot{\psi}_A \sin\psi_R \cos\psi_p \\ \dot{\psi}_A \sin\psi_p \\ \dot{\psi}_p \sin\psi_R + \dot{\psi}_A \cos\psi_R \cos\psi_p \end{Bmatrix}_{\hat{\underline{f}}} \qquad (E-2)$$

Finally add $\dot{\psi}_R \, \hat{\underline{f}}_2$ to the result of (E-2) to complete the determination of the $\{\hat{\underline{f}}\}$ components of $\underline{\omega}_f$

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$$\omega_{f1} = \dot{\psi}_p \cos\psi_R - \dot{\psi}_A \sin\psi_R \cos\psi_p$$

$$\omega_{f2} = \dot{\psi}_R + \dot{\psi}_A \sin\psi_p \tag{E-3}$$

$$\omega_{f3} = \dot{\psi}_p \sin\psi_R + \dot{\psi}_A \cos\psi_R \cos\psi_p$$

3. Transform (3-4) into $\hat{\underline{p}}$ Coordinates

Now proceed to transform all components of Eq. (3-4) into the $\{\hat{\underline{p}}\}$ frame.

Transform $\omega_{fi}$ in Eq. (E-3) into $\{\hat{\underline{f}}'\}$ using from Section 2.0

$$\{\hat{\underline{f}}'\} = [C_3(\theta_T)] \, \{\hat{\underline{f}}\}$$

$$
\begin{bmatrix} \cos\theta_T & \sin\theta_T & 0 \\ -\sin\theta_T & \cos\theta_T & 0 \\ 0 & 0 & 1 \end{bmatrix}
\begin{Bmatrix} \omega_{f1} \\ \omega_{f2} \\ \omega_{f3} \end{Bmatrix}_{\hat{\underline{f}}}
=
\begin{Bmatrix} \omega_{f1}\cos\theta_T + \omega_{f2}\sin\theta_T \\ -\omega_{f1}\sin\theta_T + \omega_{f2}\cos\theta_T \\ \omega_{f3} \end{Bmatrix}_{\hat{\underline{f}}'}
\tag{E-4}
$$

Next add $\dot{\theta}_T \, \hat{\underline{f}}'_3$ to the result of Eq. (E-4) and transform into $\{\hat{\underline{g}}\}$ using from Section 2.0

$$\{\hat{\underline{g}}\} = [C_1(\theta_x)] \, \{\hat{\underline{f}}'\}$$

We facilitate this by defining the first two terms in Eq. (E-4)

$$D_1 = \omega_{f1}\cos\theta_T + \omega_{f2}\sin\theta_T$$

$$D_2 = -\omega_{f1}\sin\theta_T + \omega_{f2}\cos\theta_T \qquad \text{(E-5)}$$

Then

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & \sin\theta_x \\ 0 & -\sin\theta_x & \cos\theta_x \end{bmatrix} \left\{ \begin{array}{c} D_1 \\ D_2 \\ \omega_{f3}+\dot{\theta}_T \end{array} \right\}_{\hat{\underline{f}}} = \left\{ \begin{array}{c} D_1 \\ D_2\cos\theta_x+(\dot{\theta}_T+\omega_{f3})\sin\theta_x \\ -D_2\sin\theta_x+(\dot{\theta}_T+\omega_{f3})\cos\theta_x \end{array} \right\}_{\hat{\underline{g}}} \text{(E-6)}$$

Next add $\dot{\theta}_x \, \hat{\underline{g}}_1$ to the result of Eq. (E-6) and transform into $\{\hat{\underline{p}}\}$ using from Section 2.0

$$\{\hat{\underline{p}}\} = [C_3(\theta_y)] \, \{\hat{\underline{g}}\}$$

First redefine the last two terms in (E-6) as

$$Y_2 = D_2\cos\theta_x + (\dot{\theta}_T + \omega_{f3})\sin\theta_x$$

$$Y_3 = -D_2\sin\theta_x + (\dot{\theta}_T + \omega_{f3})\cos\theta_x \qquad \text{(E-7)}$$

Then

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$$\begin{bmatrix} \cos\theta_y & \sin\theta_y & 0 \\ -\sin\theta_y & \cos\theta_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \left\{ \begin{matrix} D_1 + \dot\theta_x \\ y_2 \\ y_3 \end{matrix} \right\}_{\hat{\underline{q}}} = \left\{ \begin{matrix} (D_1 + \dot\theta_x)\cos\theta_y + y_2\sin\theta_y \\ -(D_1 + \dot\theta_x)\sin\theta_y + y_2\cos\theta_y \\ y_3 \end{matrix} \right\}_{\hat{\underline{p}}} \quad \text{(E-8)}$$

Finally add $\dot\theta_y\, \hat{\underline{p}}_3$ to the result of (E-8) to get

$$\left. \begin{matrix} \omega_{p1} = (D_1 + \dot\theta_x)\cos\theta_y + y_2\sin\theta_y \\[1em] \omega_{p2} = -(D_1 + \dot\theta_x)\sin\theta_y + y_2\cos\theta_y \\[1em] \omega_{p3} = y_3 + \dot\theta_y \end{matrix} \right\} \quad \text{(E-9)}$$

4.    Solution for $\dot\theta_x$, $\dot\theta_y$

Now solve directly for $\dot\theta_x$ and $\dot\theta_y$ using the third and first of Eq. (E-9)

$$\left. \begin{matrix} \dot\theta_y = \omega_{p3} - y_3 \\[1.5em] \dot\theta_x = (\omega_{p1} - D_1\cos\theta_y - y_2\sin\theta_y)/\cos\theta_y \end{matrix} \right\} \quad \text{(E-10)}$$

These are system equations (10-1) and (10-2). It is significant to note that these equations for $\dot\theta_y$ and $\dot\theta_x$ are functions of $\omega_{p1}$, $\omega_{p3}$, $\dot\theta_T$, $\omega_{f1}$, $\omega_{f2}$, $\omega_{f3}$, $\theta_x$ and $\theta_T$. Notice that $\omega_{p2}$ does not enter these calculations. We make use of this fact in Appendix G.

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## GENERATION OF SYSTEM EQUATIONS (10-5), (10-6) AND (10-7)
$$\text{FOR } \dot{\omega}_{p3}, \dot{\omega}_{p1}, \dot{\omega}_{p2}$$

### 1. Overview of Process

Considerable algebra is involved in obtaining Eqs. (10-5) and (10-6) due to the need to eliminate unknown bearing reaction torques $T_{a3}$, $T_{b1}$, $T_{pG}$. The Euler equations (6-2) and (6-3) are the starting points with additional relations coming from the constraint equations in Section 7.0. A number of transformations are required to obtain scalar equations in a common frame.

Specific steps in the process are:

a) Differentiate constraint Eqs. (7-2) and (7-3) to obtain new equations for $\dot{\omega}_{a3}$ and $\dot{\omega}_{b1}$.

b) Combine results of step a) with gyro Euler Eqs. (5-2) and (5-3) to define reaction torques $T_{a3}$ and $T_{b1}$.

c) Perform transformations to define $\hat{p}$ components of total torque $\underline{T}_p$ in Eq. (9-22).

d) Introduce $T_{pi}$ components from step c) into platform Euler equations in Section 6.0, then use results of step b) to eliminate $T_{a3}$ and $T_{b1}$.

e) Use $\dot{\omega}_{p2}$ as defined from velocity constraints in Appendix G to eliminate $T_{pG}$ leaving equations for $\dot{\omega}_{p3}$ and $\dot{\omega}_{p1}$.

### 2. Differentiate Constraint Equations

Differentiate constraint Eqs. (7-2) and (7-3) to obtain new equations for $\dot{\omega}_{a3}$ and $\dot{\omega}_{b1}$.

$$\omega_{a3} = -\omega_{p2}s^{.}\ \gamma_a + \omega_{p3}\cos\gamma_a \tag{7-2}$$

$$\omega_{b1} = \omega_{p1}\cos\gamma_b + \omega_{p2}\sin\gamma_b \tag{7-3}$$

Then

$$\dot{\omega}_{a3} = -\dot{\omega}_{p2}\sin\gamma_a + \dot{\omega}_{p3}\cos\gamma_a + U_{a3} \tag{F-1}$$

$$\dot{\omega}_{b1} = \dot{\omega}_{p1}\cos\gamma_b + \dot{\omega}_{p2}\sin\gamma_b + U_{b1} \tag{F-2}$$

where

$$U_{a3} = -\omega_{p2}\dot{\gamma}_a\cos\gamma_a - \omega_{p3}\dot{\gamma}_a\sin\gamma_a$$

$$U_{b1} = -\omega_{p1}\dot{\gamma}_b\sin\gamma_b + \omega_{p2}\dot{\gamma}_b\cos\gamma_b$$

?. Define $T_{a3}$ and $T_{b1}$

From the Euler equations in Section 5.0

$$\dot{\omega}_{a3} = J_{a3} + T_{a3}/I_{g3} \tag{5-2}$$

$$\dot{\omega}_{b1} = J_{b1} + T_{b1}/I_{g1} \tag{5-3}$$

Use (F-1), (F-2), (5-2), (5-3) to eliminate $\dot{\omega}_{a3}$ and $\dot{\omega}_{b1}$ and define $T_{a3}$ and $T_{b1}$

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$$T_{a3} = I_{g3}[-\dot{\omega}_{p2}\sin\gamma_a + \dot{\omega}_{p3}\cos\gamma_a + U_{a3} - J_{a3}] \qquad (F-3)$$

$$T_{b1} = I_{g1}[\dot{\omega}_{p1}\cos\gamma_b + \dot{\omega}_{p2}\sin\gamma_b + U_{b1} - J_{b1}] \qquad (F-4)$$

4.  Platform Torque in $\{\hat{\underline{p}}\}$ coordinates

Next express

$$\underline{T}_p = T_{p1}\,\hat{\underline{P}}_1 + T_{p2}\,\hat{\underline{P}}_2 + T_{p3}\,\hat{\underline{P}}_3 \qquad (F-5)$$

for use in the platform Euler equations, Section 6.0.

Start with Eq. (9-22) for $\underline{T}_p$ and identify the components in that equation from Section 9.0.

$$\underline{T}_p = \underline{T}_{ps} + \underline{T}_{pF} + \underline{T}_{pu} + \underline{T}_{pB} + \underline{T}_{Rp} + \underline{T}_{pc} + \underline{T}_{pG} \qquad (9-22)$$

$$\underline{T}_{ps} = -K_{sp}[\theta_x\,\hat{\underline{g}}_1 + \theta_y\,\hat{\underline{P}}_3] \qquad (9-15)$$

$$\underline{T}_{pF} = -K_{Fp}[\{\dot{\theta}_x/|\dot{\theta}_x|\}\,\hat{\underline{g}}_1 + \{\dot{\theta}_y/|\dot{\theta}_y|\}\,\hat{\underline{P}}_3] \qquad (9-16)$$

$$\underline{T}_{pu} = T_{u1}\,\hat{\underline{P}}_1 + T_{u2}\,\hat{\underline{P}}_2 + T_{u3}\,\hat{\underline{P}}_3 \qquad (9-17)$$

$$\underline{T}_{pB} = T_{Bx}\,\hat{\underline{g}}_1 + T_{By}\,\hat{\underline{P}}_3 \qquad (9-18)$$

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$$\underline{T}_{Rp} = -\underline{T}_a - \underline{T}_b \tag{9-19}$$

$$\underline{T}_{pc} = -K_{pc}(\gamma_b \,\hat{\underline{q}}_1 + \gamma_a \,\hat{\underline{p}}_3) - K_{pI}(\int\gamma_b dt \,\hat{\underline{q}}_1 + \int\gamma_a dt \,\hat{\underline{p}}_3) \tag{9-20}$$

$$\underline{T}_{pG} = T_{pG} \,\hat{\underline{q}}_2 \tag{9-21}$$

Rewrite Eq. (9-22) as

$$\underline{T}_p = T_1 \,\hat{\underline{q}}_1 + T_3 \,\hat{\underline{p}}_3 - \underline{T}_a - \underline{T}_b + T_{pG} \,\hat{\underline{q}}_2 + \underline{T}_{pu} \tag{F-6}$$

where

$$T_1 = -K_{sp}\theta_x - K_{Fp}\{\dot{\theta}_x/|\dot{\theta}_x|\} + T_{Bx} - K_{pc}\gamma_b - K_{pI}\int\gamma_b dt$$

$$T_3 = -K_{sp}\theta_y - K_{Fp}\{\dot{\theta}_y/|\dot{\theta}_y|\} + T_{By} - K_{pc}\gamma_a - K_{pI}\int\gamma_a dt$$

$$\underline{T}_a = T_{a1} \,\hat{\underline{a}}_1 + T_{a3} \,\hat{\underline{a}}_3 \quad \text{(see Eq. (9-7))}$$

$$\underline{T}_b = T_{b1} \,\hat{\underline{b}}_1 + T_{b3} \,\hat{\underline{b}}_3 \quad \text{(see Eq. (9-14))}$$

Now start a series of transformations to put all components of Eq. (F-6) into the $\{\hat{\underline{p}}\}$ frame.

First transform $\underline{T}_a = -T_{a1} \,\hat{\underline{a}}_1 - T_{a3} \,\hat{\underline{a}}_3$ into $\{\hat{\underline{p}}\}$ using, from Section 2.0

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$$\{\hat{\underline{p}}\} = [C_1(\gamma_a)]^T\{\hat{\underline{a}}\}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma_a & -\sin\gamma_a \\ 0 & \sin\gamma_a & \cos\gamma_a \end{bmatrix} \begin{Bmatrix} -T_{a1} \\ 0 \\ -T_{a3} \end{Bmatrix}_{\hat{\underline{a}}} = \begin{Bmatrix} -T_{a1} \\ T_{a3}\sin\gamma_a \\ -T_{a3}\cos\gamma_a \end{Bmatrix}_{\hat{\underline{p}}} \qquad (F-7)$$

Next transform $\underline{T}_b = -T_{b1}\,\hat{\underline{b}}_1 -T_{b3}\,\hat{\underline{b}}_3$ into $\{\hat{\underline{p}}\}$ using from Section 2.0

$$\{\hat{\underline{p}}\} = [C_3(\gamma_b)]^T\{\hat{\underline{b}}\}$$

$$\begin{bmatrix} \cos\gamma_b & -\sin\gamma_b & 0 \\ \sin\gamma_b & \cos\gamma_b & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} -T_{b1} \\ 0 \\ -T_{b3} \end{Bmatrix}_{\hat{\underline{b}}} = \begin{Bmatrix} -T_{b1}\cos\gamma_b \\ -T_{b1}\sin\gamma_b \\ -T_{b3} \end{Bmatrix}_{\hat{\underline{p}}} \qquad (F-8)$$

Next transform $T_1\,\hat{\underline{g}}_1 + T_{pG}\,\hat{\underline{g}}_2$ into $\{\hat{\underline{p}}\}$ using from Section 2.0

$$\{\hat{\underline{p}}\} = [C_3(\theta_y)]\{\hat{\underline{g}}\}$$

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$$\begin{bmatrix} \cos\theta_y & \sin\theta_y & 0 \\ -\sin\theta_y & \cos\theta_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_{pG} \\ 0 \end{Bmatrix}_{\hat{q}} = \begin{Bmatrix} T_1\cos\theta_y + T_{pG}\sin\theta_y \\ -T_1\sin\theta_y + T_{pG}\cos\theta_y \\ 0 \end{Bmatrix}_{\hat{p}} \qquad (F\text{-}9)$$

Now sum all of the $\{\hat{p}\}$ components in (F-7), (F-8), (F-9) with $T_3\,\hat{p}_3$ in (F-6) and $\underline{T}_{pu}$ as defined in (9-17) to obtain the $T_{pi}$ components in (F-5).

$$\left. \begin{aligned} T_{p1} &= -T_{a1} - T_{b1}\cos\gamma_b + T_1\cos\theta_y + T_{pG}\sin\theta_y + T_{u1} \\ T_{p2} &= T_{a3}\sin\gamma_a - T_{b1}\sin\gamma_b - T_1\sin\theta_y + T_{pG}\cos\theta_y + T_{u2} \\ T_{p3} &= -T_{a3}\cos\gamma_a - T_{b3} + T_3 + T_{u3} \end{aligned} \right\} \qquad (F\text{-}10)$$

5.  Put Terms into Euler Equations

At this point combine the definitions of $T_{a3}$ and $T_{b1}$ in (F-3) and (F-4) with the $\{\hat{p}\}$ coordinates of $\underline{T}_p$ in (F-10) into the platform Euler equations in Section 6.0.

The Euler equations are:

$$\dot{\omega}_{p1} = T_{p1}/I_{p1} \qquad (6\text{-}1)$$

$$\dot{\omega}_{p2} = T_{p2}/I_{p2} \qquad (6\text{-}2)$$

$$\dot{\omega}_{p3} = T_{p3}/I_{p3} \qquad (6\text{-}3)$$

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Using Eqs. (F-3) and (F-4) write $T_{a3}$ and $T_{b1}$ as

$$T_{a3} = r_1 \dot{\omega}_{p2} + r_2 \dot{\omega}_{p3} + r_3 \qquad\qquad\qquad (F\text{-}11)$$

$$T_{b1} = s_1 \dot{\omega}_{p1} + s_2 \dot{\omega}_{p2} + s_3 \qquad\qquad\qquad (F\text{-}12)$$

where

$$r_1 = -I_{g3}\sin\gamma_a \qquad\qquad s_1 = I_{g1}\cos\gamma_b$$

$$r_2 = I_{g3}\cos\gamma_a \qquad\qquad s_2 = I_{g1}\sin\gamma_b$$

$$r_3 = I_{g3}(U_{a3} - J_{a3}) \qquad s_3 = I_{g1}(U_{b1} - J_{b1})$$

Use (F-11) and (F-12) to eliminate $T_{a3}$ and $T_{b1}$ in Eq. (F-10) and substitute the result into Eqs. (6-1), (6-2), (6-3).

$$I_{p1}\dot{\omega}_{p1} = -T_{a1} - \cos\gamma_b(s_1\dot{\omega}_{p1} + s_2\dot{\omega}_{p2} + s_3)$$

$$+ T_1\cos\theta_y + T_{u1} + T_{pG}\sin\theta_y \qquad\qquad (F\text{-}13)$$

$$I_{p2}\dot{\omega}_{p2} = \sin\gamma_a(r_1\dot{\omega}_{p2} + r_2\dot{\omega}_{p3} + r_3)$$

$$- \sin\gamma_b(s_1\dot{\omega}_{p1} + s_2\dot{\omega}_{p2} + s_3)$$

$$- T_1\sin\theta_y + T_{u2} + T_{pG}\cos\theta_y \qquad\qquad (F\text{-}14)$$

$$I_{p3}\dot{\omega}_{p3} = -\cos\gamma_a(r_1\dot{\omega}_{p2} + r_2\dot{\omega}_{p3} + r_3) - T_{b3} + T_3 + T_{u3} \qquad (F\text{-}15)$$

6. <u>Elimination of $T_{pG}$ and Solutions</u>

Eliminate $T_{pG}$ by combining Eqs. (F-13) and (F-14) to obtain:

$$\dot{\omega}_{p1}d_5 + \dot{\omega}_{p2}d_6 + \dot{\omega}_{p3}d_7 = d_8 \qquad (F\text{-}16)$$

where
$$d_5 = s_1\sin\gamma_b \tan\theta_y - I_{p1} - s_1\cos\gamma_b$$

$$d_6 = (s_2\sin\gamma_b - r_1\sin\gamma_a + I_{p2}) \tan\theta_y - s_2\cos\gamma_b$$

$$d_7 = -r_2\sin\gamma_a \tan\theta_y$$

$$d_8 = T_{a1} + s_3\cos\gamma_b - T_1\cos\theta_y - T_{u1}$$

$$+ (r_3\sin\gamma_a - s_3\sin\gamma_b - T_1\sin\theta_y + T_{u2}) \tan\theta_y$$

Write Eq. (F-15) as:

$$\dot{\omega}_{p3} = d_{20}\dot{\omega}_{p2} + d_{21}\dot{\omega}_{p3} + d_{22} \qquad (F\text{-}17)$$

where
$$d_{20} = -(r_1\cos\gamma_a)/I_{p3}$$

$$d_{21} = -(r_2\cos\gamma_a)/I_{p3}$$

$$d_{22} = (-r_3\cos\gamma_a - T_{b3} + T_3 + T_{u3})/I_{p3}$$

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This leaves two equations ((F-16) and (F-17)) in three unknowns, $\dot{\omega}_{p1}$, $\dot{\omega}_{p2}$, $\dot{\omega}_{p3}$. Obtain a third equation from the velocity constraints due to the gimballing that is determined in Appendix G. The result is:

$$\dot{\omega}_{p2} = -\dot{\omega}_{p1} \tan\theta_y - d_{10}\sin\theta_y + d_{13} \qquad \text{(G-3)}$$

where $d_{10}$ and $d_{13}$ are defined in Appendix G

The solution of this set of equations ((F-16), (F-17)), (G-3) is:

$$\dot{\omega}_{p1} = \frac{d_{11}}{d_{12}} \qquad \text{(F-18)}$$

$$\dot{\omega}_{p2} = - \frac{d_{11}}{d_{12}} \tan\theta_y - d_{10}\sin\theta_y + d_{13} \qquad \text{(F-19)}$$

$$\dot{\omega}_{p3} = \frac{d_{20}\dot{\omega}_{p2} + d_{22}}{1 - d_{21}} \qquad \text{(F-20)}$$

where
$$d_{11} = d_8 + (d_{13} - d_{10}\sin\theta_y)(-d_6 - \frac{d_7 d_{20}}{1 - d_{21}}) - (\frac{d_7 d_{22}}{1 - d_{21}})$$

$$d_{12} = d_5 + \tan\theta_y(-d_6 - \frac{d_7 d_{20}}{1 - d_{21}})$$

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# APPENDIX G

## GENERATION OF $\dot{\omega}_{p2}$ FROM VELOCITY CONSTRAINTS

This appendix develops $\dot{\omega}_{p2}$ from velocity constraints imposed by the gimballing configuration for use in Appendix F as the third of the three platform system equations.

The process here is to use Eq. (E-9) for $\omega_{p2}$ and differentiate. Additional relations are obtained from Appendix E, noting that $\theta_x$ and $\theta_y$ are independent of $\omega_{p2}$ as indicated by Eq. (E-10).

$$\omega_{p2} = -(D_1 + \dot{\theta}_x)\sin\theta_y + y_2\cos\theta_y \qquad \text{(E-9)}$$

where

$$D_1 = \omega_{f1}\cos\theta_T + \omega_{f2}\sin\theta_T$$

$$D_2 = -\omega_{f1}\sin\theta_T + \omega_{f2}\cos\theta_T$$

$$y_2 = D_2\cos\theta_x + (\dot{\theta}_T + \omega_{f3})\sin\theta_x$$

Differentiating Eq. (E-9) produces

$$\dot{\omega}_{p2} = -(\dot{D}_1 + \ddot{\theta}_x)\sin\theta_y - \dot{\theta}_y(D_1 + \dot{\theta}_x)\cos\theta_y$$
$$+ \dot{y}_2\cos\theta_y - y_2\dot{\theta}_y\sin\theta_y \qquad \text{(G-1)}$$

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We now require relations for $\ddot{\theta}_x$, $\dot{D}_1$, $\dot{y}_2$.  From Appendix E, Eq. (E-10)

$$\dot{\theta}_x = (\omega_{p1} - D_1 \cos\theta_y - y_2 \sin\theta_y)/\cos\theta_y \qquad (E\text{-}10)$$

Differentiating Eq. (E-12) produces

$$\ddot{\theta}_x = \frac{\dot{\omega}_{p1}}{\cos\theta_y} + d_{10} \qquad (G\text{-}2)$$

where

$$d_{10} = (-\dot{D}_1 \cos\theta_y + D_1 \dot{\theta}_y \sin\theta_y - \dot{y}_2 \sin\theta_y - y_2 \dot{\theta}_y \cos\theta_y)$$

$$(\cos^{-1}\theta_y) + \dot{\theta}_y(\omega_{p1} - D_1 \cos\theta_y - y_2 \sin\theta_y)$$

$$\cos^{-2}\theta_y \sin\theta_y$$

Substitute Eq. (G-2) into (G-1) to obtain the solution

$$\dot{\omega}_{p2} = -\dot{\omega}_{p1}\tan\theta_y - d_{10}\sin\theta_y + d_{13} \qquad (G\text{-}3)$$

where $\quad d_{13} = -\dot{D}_1 \sin\theta_y - \dot{\theta}_y(D_1 + \dot{\theta}_x)\cos\theta_y$

$$+ \dot{y}_2 \cos\theta_y - y_2 \dot{\theta}_y \sin\theta_y$$

We define $\dot{D}_1$, $\dot{D}_2$ and $\dot{y}_2$ by differentiating the equations under (E-9) above:

$$\dot{D}_1 = \dot{\omega}_{f1}\cos\theta_T - \omega_{f1}\dot{\theta}_T\sin\theta_T + \dot{\omega}_{f2}\sin\theta_T + \omega_{f2}\dot{\theta}_T\cos\theta_T \qquad (G-4)$$

$$\dot{D}_2 = -\dot{\omega}_{f1}\sin\theta_T - \omega_{f1}\dot{\theta}_1\cos\theta_T + \dot{\omega}_{f2}\cos\theta_T - \omega_{f2}\dot{\theta}_T\sin\theta_T \qquad (G-5)$$

$$\dot{y}_2 = \dot{D}_2\cos\theta_x - D_2\dot{\theta}_x\sin\theta_x$$

$$+ (\ddot{\theta}_T + \dot{\omega}_{f3})\sin\theta_x + \dot{\theta}_x(\dot{\theta}_T + \omega_{f3})\cos\theta_x \qquad (G-6)$$

We define the $\dot{\omega}_{fi}$ terms by differentiating Eqs. (E-3) for $\omega_{fi}$ in Appendix E.

$$\dot{\omega}_{f1} = \ddot{\psi}_p\cos\psi_R - \dot{\psi}_p\dot{\psi}_R\sin\psi_R - \ddot{\psi}_A\sin\psi_R\cos\psi_p - \dot{\psi}_A\dot{\psi}_R\cos\psi_R\cos\psi_p$$

$$+ \dot{\psi}_A\dot{\psi}_p\sin\psi_R\sin\psi_p \qquad (G-7)$$

$$\dot{\omega}_{f2} = \ddot{\psi}_R + \ddot{\psi}_A\sin\psi_p + \dot{\psi}_A\dot{\psi}_p\cos\psi_p \qquad (G-8)$$

$$\dot{\omega}_{f3} = \ddot{\psi}_p\sin\psi_R + \dot{\psi}_p\dot{\psi}_R\cos\psi_R + \ddot{\psi}_A\cos\psi_R\cos\psi_p$$

$$- \dot{\psi}_A\dot{\psi}_R\sin\psi_R\cos\psi_p - \dot{\psi}_A\dot{\psi}_p\cos\psi_R\sin\psi_p \qquad (G-9)$$

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# APPENDIX H

## COMPUTER PROGRAMS

### INTRODUCTION

The Fortran computer programs contain the inputs, torques and system equations defined in this report and computes the system trajectory by use of Runge Kutta integration.

### PROGRAMS AND SUBROUTINES

| | |
|---|---|
| MT21D: | Main program |
| MTRK1: | 4-cycle Runge Kutta integration |
| MT22: | Ship motion and coordinate transformation for command inputs |
| MT19B: | Torques |
| MT20B: | State equations |
| PLOT: | Library plotting routine |

Figure H-1 is an overall block representation of the program. The listing of the program is attached.

```
                    ┌─────────────────┐
                    │  TR-562(H-1)    │
                    └─────────────────┘
                    ┌─────────────────┐
                    │     MT21 D      │
                    └─────────────────┘
                             │
    ┌────────────────────────┼───────┐
    │   ┌────────────────┐    │       │
    │   │           ┌────▼────▼───────▼──┐
    │   │           │      MTRK 1        │
    │   │           └─────────┬──────────┘
    │   │           ┌─────────▼──────────┐
    │   │           │      MT 22         │
    │   │           │      MT 19B        │
    │   │           │      MT 20B        │
    │   │           └─────────┬──────────┘
    │   │                     │
    │   │                   ╱   ╲
    │   └──────────────────◁ OUTPUT ▷
    │                       ╲ DECISION ╱
    │                        ╲(TMAX) ╱
    │                           │
    │                 ┌─────────▼──────────┐
    │                 │     STORE FOR      │
    │                 │     PLOTTING       │
    │                 └─────────┬──────────┘
    │                         ╱   ╲
    │                        ╱     ╲
    └───────────────────────◁ TERMINATE ▷
                            ╲ DECISION ╱
                             ╲(TEND) ╱
                                │
                      ┌─────────▼──────────┐
                      │       PLOT         │
                      └────────────────────┘
```
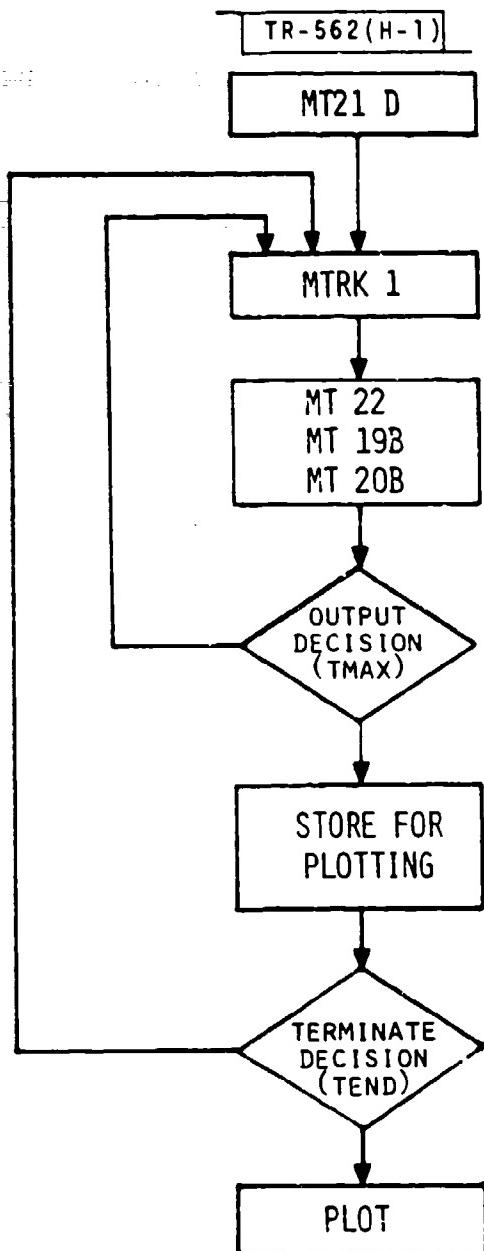
Fig. H-1.   Computer flow diagram.

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```
    MT21D

C  B AXIS DMS PLATFORM
C  TRAINABLE X-Y PLATFORM
C  RT19D CONTAINS ,DYNAMICS, TORQUES
C  RT29B CONTAINS SYSTEM EQUATIONS
C  RT2B CONTAINS GEOMETRY,SHIP MOTION
C
C  SYSTEM VARIABLES
C
C  XI(1)-THETA ELEVATION OUT      XI(5)-UP3
C  XI(8)-THETA CROSS ELEV OUT     XI(6)-UP1
C  XI(3)-UA1                      XI(7)-UP2
C  XI(4)-UB3                      XI(10)-GAMMA A
C                                 XI(9)-GAMMA B
C  XI(18)-GAMMA A A INTEGR        XI(11)-GAMMA B INTEGR
C  XI(12)-THETA TRAIN             XI(13)-THETA DOT TRAIN
C
C  D(1)-DESIRED AZIM(DEG)         D(8)-FLEX PERIOD(SEC)
C  D(2)-DESIRED ELEV(DEG)         D(9)-ROLL HEIGHT(IN)
C  D(3)-ROLL AMPLIT(DEG)          D(10)-VELOCITY(KNOTS)
C  D(4)-PITCH AMPLIT(DEG)         D(11)-TURN RATE(DEG/SEC)
C  D(5)-FLEX AMPLIT(DEG)          E(12)-HEEL ANGLE(DEG)
C  D(6)-ROLL PERIOD(SEC)          D(13)-CANT ANGLE(DEG)
C  D(7)-PITCH PERIOD(SEC)         D(14)-UM TRAIN SERVO(RAD/SEC)
C                                 D(15)-DAMPING RATIO TRAIN SERVO
C
C  E(1)-PLATFORM MASS(LB-SEC2/IN)
C  E(2)-GYRO MASS
C  E(3)-GYRO MASS OFFSET(IN)
C  E(4)- PLAT 1 MASS OFFSET(IN)
C  E(5)- PLATFORM 2 MASS OFFSET(IN)
C  E(6)- PLATFORM 3 MASS OFFSET(IN)
C  E(7)-GYRO FRICT.(IN-LB)
C  E(8)-GYRO SPRING(IN-LB/RAD)
C  E(9)-GYRO A CONTROL GAIN(IN-LB/RAD)
C  E(10)-GYRO B CONTROL GAIN(IN-LB/RAD)
C  E(11)-PLAT FRICT(IN-LB)
C  E(12)-PLATFORM SPRING(IN-LB/RAD)
C  E(13)-GYRO INERTIA 1 AND 3(LB-IN-SEC2)
C  E(14)-GYRO INERTIA 2
C  E(15)- PLATFORM INERTIA 1
C  E(16)-PLATFORM INERTIA 2
C  E(17)-PLATFORM INERTIA 3
C  E(18)-GYRO SPIN SPEED(RAD/SEC)
C  E(19)-PLATF CONTR GAIN(IN-LB/RAD)
C  E(20)-PLATF TORQ INTEGR GAIN(IN-LB/RAD-SEC)
C  E(21)-GYRO A TORQUE BIAS(IN-LB)
C  E(22)-GYRO B TORQUE BIAS(IN-LB)
C  E(23)-PLAT ELEV BIAS TORQUE(IN-LB)
C  E(24)-PLAT CROSS ELEV BIAS(IN-LB)
C  E(25)-LIMIT ON GYRO TORQUE(IN-LB)
C  E(26)-GYRO SPEED MISMATCH
C  SPEED OF GYRO A=E(18)E(1+D0+.5D0E(26))
C  SPEED OF GYRO B=E(18)E(1-D0+.5D0E(26))
C
C  G(1)-TPCE          G(8)-UF2
C  G(2)-TPE           G(9)-UF3
C  G(3)-TB3           G(10)-TU1
C  G(4)-TA1           G(11)-TU2
C  G(5)-TB1           G(12)-TU3
C  G(6)-TCE1          G(13)-UFD1
C  G(7)-UF1           G(14)-UFDB

C  G(15)-UFD3
C
      IMPLICIT REAL*8 (A-H,O-Z)
      NAMELIST/INPUTS/TEND,TMAX,DT,XI,DXDT,D,E,TI,TITRM,SYPLT
      DIMENSION XI(13),XU(13),DXDT(13),CDUM(65),D(15),
     1 MP(3),B(3),CR(3,3),CP(3,3),CT(3,3),CA(3,3),BU(3),
     1 S(2),CE(3,3),CI(3,3),C1(3,3),C2(3,3),
     1 A(3),G(26),E(26),RUNNAM(10)
C
      REAL*8 NP
      DATA XI/13*0.D0/
      DATA DXDT/130.D0/
      DATA D/6.D0, 0.D0, 15.D0, 15.D0, 10.D0, 15.D0, 1.D0,
     1     1200.D0, 0.D0, 0.D0, 0.D0, 3.D0, 0.D0, 700.D0,
      DATA E/.25D0, .05D0, .003D0, .003D0, .003D0, .003D0,
     1     .04D0, .2D0, 108.D0, .5D0, 1.5D0, .5D0, 1.D0,
     1     40.D0, 30.D0, 40.D0, 361.D0, 50.D0, 5.DJ,0.D0,0.D0,0.
      D0.5
C
      TEND-30.D0
      DT-.005D0
      SYPLT-0.D0
      TITRM-200.D0
      TI-0.D0
      TCAL-0.D0
      TMAX-.25D0
C
C  READ RUN NAM AND PARAMETERS TO CHANGE
5555  READ(1,5555) (RUNNAM(K),K=1,10)
      FORMAT(10A4)
      READ(1,INPUTS)
      ADD-.25D0
      WRITE(6,5556) (RUNNAM(K),K=1,10)
5556  FORMAT(1X,10A4//)
C
C  WRITE RUN NAME TO PLOT FILE
      WRITE(16,5557) (RUNNAM(K),K=1,10)
5557  FORMAT('RANN DOT ',10A4)
C
      WRITE(16,66)
66    FORMAT('*COL /1X, T(SEC    CER(DEG    EER(DEG    GA(DEG
     1 (DEG     TB3(IL     TA1(IL     UP1(D,SEC    .1XUP2(D,SEC
      UP3(D,SEC')
C
      L=1
1     IF(TI.GT.TITRM)D(1)=3.D0
2     CALL MTM1(G5,L,13,DXDT,XI,TI,DT,XU,TU,CDUM)
      IF(L-6)3,4,3
3     CONTINUE
      CALL MT2B(D,TI,XI,DXDT,MP,B,CR,CP,CA,BU,S,
     1     CE,CT,CI,C2,A,G,E)
C
      CALL MT19B(D,TI,XI,DXDT,A,G,E,SUR,DY,TAOLD,TBOLD)
      CALL MT29B(XU,DXDT,Q,E,D,TU)
      GO TO 2
4     CONTINUE
      IF(TMAX-TI)6,6,1
5     CONTINUE
C
```

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```
                                                    TU=TI+DT*.5D0
                                                    L=3
                                                    RETURN
                                                C
                                              3    DO 7 J=1,M
                                                    I=M+J
                                                    C(I)=DXDT(J)*DT
                                              7    XU(J)=XI(J)+C(I)*.5D0
                                                    L=4
                                                    RETURN
                                                C
                                              4    DO 8 J=1,M
                                                    J=2M+J
                                                    C(I)=DXDT(J)*DT
                                              8    XU(J)=XI(J)+C(I)
                                                    TU=TI+DT
                                                    L=5
                                                    RETURN
                                                C
                                              5    DO 34 J=1,M
                                                    I=3M+J
                                              34   C(I)=DXDT(J)*DT
                                                C
                                                C COMPUTE ANSWER
                                              21   DO 17 J=1,M
                                                    M2=M+J
                                                    M3=2M+J
                                                    M4=3M+J
                                              17   XI(J)=XI(J)+(C(J)+2.D0*(C(M2)+C(M3))+C(M4))/6.D0
                                                    TI=TI+DT
                                                C
                                              18   L=6
                                                    RETURN
                                                    END
                                                    SUBROUTINE RT22(D,TU,XU,DXDT,MP,B,CR,CP,
                                                   1  CA,BU,S,CE,CT,C1,C2,A,G,E)
                                                    IMPLICIT REAL*8 (A-H,O-Z)
                                                C
                                                C SHIP MOTION,TRANSFORMATIONS,ACCELERATION,TORQUES
                                                C FOR 2 AXIS DMS STABILIZATION
                                                    DIMENSION D(15),MP(3),B(3),CR(3,3),CP(3,3),
                                                   1  CA(3,3),BU(3),S(3),V(13),CE(3,3),C(3,3),
                                                   1  C1(3,3),C2(3,3),A(3),DYDT(13),G(2B),E(2G)
                                                    REAL*8 MP
                                                C
                                                C ROLL POSITION(ANGULAR)
                                                    VR=G.28D0/D(B)
                                                    SR=(D(3)/57.2957BD0)*DSIN(URBTU)
                                                    SRC=SP+(D(13)+D(12))/57.2957BD0
                                                C ROLL ACCELERATION IN M1 PRIME
                                                    SRD=URBURZSR
                                                    AR=D(8)*BPD
                                                C ROLL VELOCITY(ANGULAR)
                                                    BUR=URB(D(3)/57.2957BD0)*BDCOS(URBTU)
                                                C PITCH POSITION(ANGULAR)
                                                    UP=G.28D0/D(7)
                                                    SP=(D(4)/57.2957BD0)*DSIN(UPBTU)
                                                C PITCH ACCELERATION IN M2 PRIME
                                                    BPD=UPBUPBSP
                                                    AP=D(B)*BPD
                                                C PITCH VELOCITY(ANGULAR)
```

```
                                                    CALL RTBR(B,TI,XI,DXDT,MP,B,CR,CP,CA,BU,S,
                                                   1  CE,CT,C1,C2,A,G,E)
                                                    V1=(G(6)-H(B))*57.2957BD0
                                                    V2=(G(8)-H(5))*57.2957BD0
                                                    V3=H(1)*B87.2957B0
                                                    V4=H2(1)*B87.2957B0
                                                    V5=H(4)*B87.2957B0
                                                    V6=.1B0*H(7)*B87.2957B0
                                                    V7=H(5)*57.2957B0
                                                    V8=G(2)
                                                    V9=G(4)
                                                C
                                                    IF(TI.LT.BPLT)GO TO B71
                                                C STORE FOR PLOTTING ...............
                                                C
                                              B7   WRITE(16,B7)V1,V1,V2,V5,V4,V8,V8,V6,V7
                                                    FORMAT(1X,1B(1X,D10.4))
                                                C
                                              B71  CONTINUE
                                                    TMAX=TMAX+BD
                                                    IF(TEND-TI)6,6,1
                                              6    CONTINUE
                                                C
                                                C COMPUTATION COMPLETED......PLOT STORED RESULTS
                                                    WRITE(15,6B8)
                                              6B8  FORMAT('BSELECT -3.,-2.,''BSELECT CER:DEG.'/
                                                   1  'BSELECT GB:DEG.'/'BPLOT')
                                                C
                                                    WRITE(16,6B9)
                                              6B9  FORMAT('BSELECT -2.,-2.,''BSELECT CA:DEG./
                                                   1  ''BSELECT GB:DEG.'/'BPLOT')
                                                C
                                                    WRITE(16,7B1)
                                              7B1  FORMAT('BSELECT -2.,-2.,''BSELECT TB3(1L''/'BSELECT TA1(1L
                                                   1  ''BPLOT')
                                                C
                                                    WRITE(16,7B8)
                                              7B8  FORMAT('BSELECT -1.,-1.,''BSELECT UP1(B,SEC''/'BSELECT -1XU
                                                   1  ''BSELECT UP3(D,SEC''/'BPLOT')
                                                    STOP
                                                    END
                                                    SUBROUTINE RTB1(N6,L,M,DXDT,XI,TI,DT,XU,TU,C)
                                                    IMPLICIT REAL*8 (A-H,O-Z)
                                                C
                                                C CYCLE RUNGE-KUTTA INTEGR TIUN.
                                                C
                                                    DIMENSION C(MB),XI(M),XU(M),DXDT(M)
                                                C
                                              33   DO TO (1,2,3,4,5),L
                                                C
                                              1    DO 20 I=1,M
                                              20   XU(I)=XI(I)
                                                    TU=TI
                                                    L=2
                                                    RETURN
                                                C
                                              2    DO 6 J=1,M
                                                    C(J)=DXDT(J)BDT
                                              6    XU(I)=XI(J)=C(J)B.5D0
```

```
C     ACCELERATION IN M1 PRIME
      AT=-D(10)XD0.DSID02D(11)/57.2957BD0
C     TOTAL ACCEL VECTOR IN M PRIME
      MP(1)=-AB + AT
      MP(2)=-AP
      MP(3)=-300.D0
C
C     AZIMUTH POSITION
      SA=D(11)XTU/57.2957BD0
C
C     SET UP FOR TRANSFORMATIONS
C     GENERATE DESIRED CROSS ELEU-ELEU TO ACHEIVE AZ-ELEU VECTOR
      B(1)=-BSIN(B(1)/57.2957BD0)XBCOS(D(2)/57.2957BD0)
      B(2)=-DCOS(B(1)/57.2957BD0)XBCOS(D(2)/57.2957BD0)
      B(3)=-DSIN(B(2)/57.2957BD0)
      CALL DCMAT(3,XU(12),CT)
      CALL DCMAT(2,SRC,CR)
      CALL DCMAT(1,SP,CP)
      CALL DCMAT(3,SA,CA)
C
      CALL RTRV3(CA,B,BU,3,3,1)
      CALL RTRV3(CP,BU,B,3,3,1)
      CALL RTRV3(CR,B,BU,3,3,1)
      CALL RTRV3(CT,BU,S,3,3,1)
C     THIS THE DESIRED CROSS ELEU - ELEU
      TCEI=-DARSIN(-S(1))
      TET=-DARSIN(S(3)/DCOS(TCE'))
C     TRANSFORM ACCEL UECTOR FROM MPRIME TO P
      CALL DCMAT(1,XU(1),CE)
      CALL DCMAT(3,XU(2),C1)
      CALL RTRV3(C1,CE,C2,3,3,3)
      CALL RTRV3(C2,CT,C1,3,3,3)
      CALL RTRV3(C1,CM,C2,3,3,3)
      CALL RTRV3(C2,CP,C1,3,3,3)
      CALL RTRV3(C1,CR,C2,3,3,3)
C     THE UECTOR A IS ACCEL IN P FROM
      CALL RTRV3(C1,MP,A,3,3,1)
C
C     FORM UF ELEMENTS, DECK VELOCITY COMPONENTS
      UF1=SUPXDCOS(SRC)-(D(11)/57.2957BD0)XDSIN(SP)XDSIN(SRC)
      UF2=SUR + (D(11)/57.2957BD0)XDSIN(SP)
      UF3=SUPXDSIN(SRC) +(B(11)/57.2957BD0)XDCOS(SP)XDCOS(SRC)
C
C     DECK ANGULAR ACCEL. UFD COMPONENTS
      UFD1 = SPXDCOS(SRC) - CUPS SUR X DSIN(SRC)
      UFD2=SPXDSIN(SRC) + SUPXSURXDCOS(SRC)
     2      -(B(11)/57.2957BD0)X(SURXDSIN(SRC)XDCOS(SP)
     1      -(B(11)/57.2957BD0)X(SURXDSIN(SRC)XDSIN(SP)
     3      +SPXBCOS(SRC)XDSIN(SP))
C     FORM OUTPUT UECTOR
      G(6)=TEI
      G(6)=TCEI
      G(7)=UF1
      G(8)=UF3
      G(13)=UFB1
      G(14)=UFD8
      G(15)=UFD2
```

```
C     G(16) FOR USE IN 104
      G(16)=SUR
      VAL1=DTAN(D(2)/57.2957BD0)XBCOS(SRC)
C
      ELSP=DATAN(VAL1)
      VAL2=DCOS(D(2)/57.2957BD0)/DCOS(ELSP)
      ELSP=DATAN(VAL1)
      CESP=DARCOS(VAL2)
      G(19)=ELSP
      G(20)=CESP
      RETURN
      END
      SUBROUTINE RT19D(D,TU,XU,DXDT,
     1    A,G,E,SUR,DT,TAOLD,TBOLD)
      IMPLICIT REAL*8 (A-M,O-Z)
C
C     FOR 2 AXIS DRS STABILIZATION
      DIMENSION D(15),A(3),DXDT(13),XU(13),G(20),E(26)
      REAL*8 MP
C
C     SKIP FLEXURE
      SF=(D(5)/57.2957BD0)XBDSIN(6.28D0XTU/D(8))
C
C     TORQUE EQUATIONS
C
C     GYRO 'A' CONTROL,BIAS,FRICTION,SPRING,UNBAL,TOTAL
      TCA=-E(9)X(G(6)-XU(2)+SF)
      IF(DABS(TCA-TAOLD).LT.3.)GO TO 20
      TCA=TAOLD+(DT/5.)X(TCA-TAOLD)
      IF(DABS(TCA).GT.E(25))TCA-E(26)X(TCA/DABS(TCA))
      TAOLD=TCA
      GDA=XU(3)-XU(6)
      IF(GDA.EQ.0.D0)GO TO 10
      YFA=-E(7)X(GDA/DABS(GDA))
      GO TO 11
   10 TFA=0.D0
   11 CONTINUE
      TSA=-E(3)XXU(8)
      YUA=-E(2)X(-A(2)XDSIN(XU(B))
     1     -A(3)XF-CS(XU(8)))
      TA1=TCA+TFA+TSA+TUA+E(21)
C
C     GYRO 'B' CONTROL,FRICT,SPRING,UNBAL,BIAS,TOTAL
      TCB=-E(10)X(G(5)-XU(1)+SF)XBDCOS(XU(2))
      IF(DABS(TCB-TBOLD).LT.3.)GO TO 21
      TCB=TBOLD + (DT/5.)X(TCB - TBOLD)
      IF(DABS(TCB).GT.E(25))TCB-E(26)X(TCB/DABS(TCB))
      TBOLD=TCB
      GDB= XU(4)-XU(5)
      IF(GDB.EQ.0.D0)GO TO 12
      TFB=-E(F7)X(GDB/DABS(GDB))
   12 TFB=0.13
   13 TFB=TCB+TFB+TCB+TUB
      ...E(8)XXU(9)
C
C     PLATFORM UNBALANCE IN P FRAME
      TU1=-E(1)XC(A(3)XE(5)-A(3)XE(5))
      TU2=-E(1)XC(A(3)XE(4)-A(1)XE(6))
      TU3=-E(1)XC(A(1)XE(6)-A(6)XE(4))
C     PLATFORM ELEVATION - FRICTION,SPRING, CONTROL,BIAS, TOTAL
```

```
      .EQ.0.D0)GO TO 16
      TPE=-E((1)3(DXDT(1)/DABS(DXDT(1)))
      GO TO 17
   16 TPE=0.D0
   17 CONTINUE
      TELCON=-E((1D)3XU(D)-E(20)3XU((1)
      T8E=-E((18)3DU((1)
      TPE=TPE+TE+TELCON+E((22)
C PLATFORM CROSS ELEV _ FRICTION.SPRING.CONTROL.BIAS.TOTAL
      IF(DMDT(2).EQ.0.D0)GO TO 14
      TPCE=-E((1)3(DXDT(2)/D/0*1*(DXDT(2)))
      GO TO 15
   14 TPCE=0.D0
   15 CONTINUE
      TSCE=-E((1)3XU(2)
      YCECON=-E((1)3XU(8) - E(20)3XU(10)
      TPCE=TPCE+TSCE+TCECON+E(24)
C
C FORM OUTPUT VECTOR
      G((1)=TPCE
      G(2)=TPE
      G(3)=-C3
      G(4)=TA1
      G(10)=TU1
      G((1)=TU2
      G(12)=TU3

      RETURN
      END
      SUBROUTINE RT20D(XU,DXDT,G,E,D,TU)
      IMPLICIT REAL3B (A-M,O-Z)
      DIMENSION XU(13),DXDT(13),G(28),E(26),D(15)
C STATE EQUATIONS FOR INTEGRATION
C TRAINABLE X-V SYSTEM
      SX1=DSIN(XU((1)
      CX1=DCOS(XU((1)
      SX2=DSIN(XU(2))
      CX2=DCOS(XU(2))
      SX8=DCOS(XU(8))
      CX8=DCOS(XU(8))
      SX9=DSIN(XU(9))
      CX9=DCOS(XU(9))
      SX12=DSIN(XU(12))
      CX12=DCOS(XU(12))
      C1=G(7)3CX12+G(8)3SX12
      C2=-G(7)3SX12+G(8)3CX12
      B2=C23CX1+(XU((1)+G(9))3SX1
C
C TRAIN SERVO CONTROL
      G4=B((1)3TU/67.2957806
      DXDT(12)=XU((1)
      DXDT((1)=-2.D0D((14)3B((1)3C3XU((1))+B((1)/67.2957DO)+D((14)33B3
      1  (B((1)/67.8957DO-XU((1)-3A)
      UA3=-XU(7)3G4SX8-XU(8)3CX8
      UA2=XU(7)3CX8-XU(8)3SX8
      UB2=-XU(8)3SX8-XU(7)3CX8
      UB1=-XU(8)3CX8+XU(7)3SX8
      DXDT(3)=-UA3SE((18)X(1.D0-.5D0SE(26))3U-UA33UB+B(4)/E((13)
      DXDT(4)=-UB13UB3E((18)X(1.D0-.5D0SE(26))3U+UB13UB3+B(3)/E((13)
C
      GDA=XU(3)-XU(8)
```

```
      GDB=XU(4)-XU(8)
      UA1=-XU(7)3GDA3CX8-XU(5)3GDA3SX8
      UB1=-XU(6)3GDB3SX9-XU(7)3GDB3CX8
      AJ3=-XU(3)3E((18)3(1.D0-.5D03E(26))3U+XU(3)3UA2
      BJ1=-XU(4)3E((18)3(1.D0-.5D0-.5D03E(26))3U-XU(4)3UB2
C
      S1=E((1)3CX9
      S2=E((1)3SX9
      S3=E((1)3(U31-BJ1)
C
      CD1=G((1)3CX12-G(7)3XU((1))3SX12+G((14)3SX12
      1  +G(8)3XU((1)3CX12
      CB2=-G((1)3SX12-G(7)3XU((1)3CX12-G((14)3CX12
      1  -G(8)3XU((1)3SX12
      DXDT((1)=-XU(6)-C13CX2-D23SX2)/CX2
      XU(7)=-(C13DXDT((1)3SX2 + D23CX2
      VD2=C23CX1-C23DXDT((1)3SX1
      1  +DXDT((1)3(XU((1)+G(9))3CX1
      Y3=-C23SX1+XU((1)3G(9))3CX1
   2  DXDT(2)=XU(5)-Y3
C
      R1=E((1)3SX8
      R2=E((1)3CX8
      R3=E((1)3(UA3-AJ3)
C
      D5=-S13SX83DTAN(XU(2))-E((1)5) - S13CX9
      D6=DTAN(XU(2))3(E((16)-R13SX8+S23SX9)
      1  -S23CX8
      D7=-R23SX83DTAN(XU(2))
      D8=Q(4)+S33CX9-G(2)3CX2-G((1))-DTAN(XU(2))
      1  3(-R33SX83S33SX93G(2)3SX2-G((1))
C
      D10=-CD1 - D23DXDT(2) + DTAN(XU(2))3
      1        (C13DXDT(2) - VD2 + (XU(6) - C13CX2 - D2 3SX2)
      2        3DXDT(2)3SX2/(CX23B2)
      D20=-R13CX8/E((17)
      D21=-R23CX8/E((17)
      D67= D6 + D73D20/(1.D0-D21)
      D12= D5 - D63DTAN(XU(2))
      D13=SX23(-CD1-D23DXDT(2))+CX23
      1        (VD2-DXDT(2)3(C1+DXDT((1)))
C
      D22=(-R33CX8-G(3)+G((1)+G(12))/E((17)
      D11=D8 - (D13 - D103SX2)3D67 - D73D22/(1.D0-D21)
      DXDY(6) = D11/D12
      DXDT(7) = -DXDT(G)3DTAN(XU(2)) + D13 - D103SX2
      DXDT(5) = -D23DXDT(7) + D22/(1.D0 - D21)
      DXDT(8)=GDA
      DXDT(9)=GDB
      DXDT((10)=-XU(8)
      DXDT((1)=-XU(9)
C SIMPLE CALCULATION FOR DXDT((1) AND DXDT(20 IN 104 B
      CEDOT=G((18)3SX1
      ELDOT=-G((16)3CX13DTAN(XU(2))
      G((17)=CEDOT
      G((18)=ELDOT
      RETURN
      END
      SUBROUTINE MTRV3(A,B,C,I,K,J)
      IMPLICIT REAL3B (A-M,O-Z)
      DIMENSION A(I,K),B(K,J), C(I,J)
      DO 10 IU=1,I
```

```
20 DO JU=1,J
   CP=0.D0
20 KU=1,K
   G(IU,JU)=G(IU,JU)+B(KU,IU))*B(KU,JU))*CP
   CP=C(IU,JU)
   CONTINUE
   CONTINUE
   RETURN
   END

C     SUBROUTINE DCMAT(N,TH,C)
      IMPLICIT REAL*8 (A-H,O-Z)
C     THIS FILLS IN THE DIRECTION COSINE MATRIX, C=MATRIX
C     N=AXIS OF ROTATION.  TH=ANGLE OF ROTATION, C=MATRIX
C
      DIMENSION C(3,3)
C
      DO 1 J=1,3
      C(J,J)=DCOS(TH)
      C(N,J)=0.D0
      C(J,N)=0.D0
      C(N,N)=1.D0
1     CONTINUE
      IF(N.EQ.1) C(2,3)=-DSIN(TH)
      IF(N.EQ.1) C(3,2)=-DSIN(TH)
C
      IF(N.EQ.2) C(1,3)=-DSIN(TH)
      IF(N.EQ.2) C(3,1)=-DSIN(TH)
C
      IF(N.EQ.3) C(2,1)=-DSIN(TH)
      IF(N.EQ.3) C(1,2)=-DSIN(TH)
      RETURN
      END
```

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# NOMENCLATURE[*]

$\underline{A}$ = total linear acceleration vector (App. D)

$A_i$ = $\hat{\underline{p}}_i$ components of $\underline{A}$

$\underline{A}_G$ = acceleration term due to gravity

$\underline{A}_p$ = fore-aft acceleration

$\underline{A}_R$ = athwartship acceleration

$\underline{A}_T$ = centripetal acceleration due to turn

$\underline{F}$ = force applied to platform at support point (App. D)

$H$ = height above center of roll

$\underline{\underline{I}}_g$ = gyro inertia diadic

$I_{gi}$ = $\hat{\underline{a}}_i$ and $\hat{\underline{b}}_i$ principal inertias of $\underline{\underline{I}}_g$

$\underline{\underline{I}}_p$ = platform inertia diadic

$I_{pi}$ = $\hat{\underline{p}}_i$ principal inertias of $\underline{\underline{I}}_p$

$K_a$ = control gain on 'A' gyro for Y axis error

$K_b$ = control gain on 'B' gyro for X axis error

$K_{FG}$ = gyro friction torque magnitude

$K_{FP}$ = platform friction torque magnitude

$K_{pc}$ = controller gain for X and Y axis torquers

$K_{pI}$ = integrator control gain for X and Y axis torquers

$K_{SG}$ = gyro spring gradient

$K_{SP}$ = platform spring gradient

$\underline{L}$ = torque vector due to unbalance

---

[*] ( $\underline{\ }$ ) denotes a vector

subscript i applies to all integers except for $d_i$

$\ell_g$ = displacement between center of support and mass for gyro

$\ell_i$ = displacement between center of support and mass for platform along $\hat{\underline{p}}_i$

M = mass of stabilized platform assembly

m = mass of gyro rotor and motor assembly

$\underline{R}$ = position vector from support point to mass center

t = time

$\underline{T}_a$ = total torque vector action on 'A' gyro

$T_{ai}$ = $\hat{\underline{a}}_i$ components of $\underline{T}_a$

$\underline{T}_{aB}$ = bias torque vector acting on 'A' gyro

$T_{aB}$ = magnitude of $\underline{T}_{aB}$

$\underline{T}_{aF}$ = friction torque acting on 'A' gyro

$\underline{T}_{aS}$ = spring torque acting on 'A' gyro

$\underline{T}_{au}$ = unbalance torque acting on 'A' gyro

$\underline{T}_b$ = total torque vector acting on 'B' gyro

$T_{bi}$ = $\hat{\underline{b}}_i$ components of $\underline{T}_b$

$\underline{T}_{bB}$ = bias torque acting on 'B' gyro

$\underline{T}_{bF}$ = friction torque acting on 'B' gyro

$\underline{T}_{bS}$ = spring torque acting on 'B' gyro

$\underline{T}_{bu}$ = unbalance torque acting on 'B' gyro

$T_{Bx}$ = x component of bias torque acting on platform

$T_{By}$ = y component of bias torque acting on platform

$\underline{T}_{ca}$ = control torque applied to 'A' gyro

$\underline{T}_{cb}$ = control torque applied to 'B' gyro

$\underline{T}_p$ = torque vector acting on platform

$T_{pi}$ = $\hat{\underline{p}}_i$ components of $\underline{T}_p$

$\underline{T}_{pB}$ = bias torque vector acting on platform

$\underline{T}_{pc}$ = control torque acting on platform

$\underline{T}_{pF}$ = friction torque acting on platform

$\underline{T}_{pG}$ = gimbal bearing reaction torque on platform

$T_{pG}$ = magnitude of $\underline{T}_{pG}$

$\underline{T}_{pS}$ = spring torque acting on platform

$\underline{T}_{pu}$ = unbalance torque acting on platform

$\underline{T}_{Ra}$ = pivot bearing reaction torque on 'A' gyro

$\underline{T}_{Rb}$ = pivot bearing reaction torque on 'B' gyro

$T_{ui}$ = $\hat{\underline{p}}_i$ components of $\underline{T}_{pu}$

$V$ = ship's speed

## GREEK LETTERS

$\alpha_A$ = satellite azimuth ephemeris

$\alpha_E$ = satellite elevation ephemeris

$\gamma_a$ = $\{\hat{\underline{a}}\}$ frame rotation angle with respect to $\{\hat{\underline{p}}\}$

$\gamma_b$ = $\{\hat{\underline{b}}\}$ frame rotation angle with respect to $\{\hat{\underline{p}}\}$

$\psi_A$ = ship's azimuth

$\psi_H$ = heel angle of deck

$\psi_p$ = pitch angle of deck

$\psi_{po}$ = amplitude of harmonic pitch motion

$\psi_R$ = roll angle of deck

$$\psi_{RH} = \psi_R + \psi_H$$

$\psi_{Ro}$ = amplitude of harmonic roll motion

$\theta_F$ = ship flexure angle

$\theta_T$ = train axis angular displacement

$\theta_x$ = X axis angular displacement

$\theta_{xc}$ = command value for $\theta_x$

$\theta_y$ = Y axis angular displacement

$\theta_{yc}$ = command value for $\theta_y$

$\underline{\omega}_a$ = angular velocity of $\{\underline{\hat{a}}\}$ frame

$\omega_{ai}$ = $\underline{\hat{a}}_i$ components of $\underline{\omega}_a$

$\underline{\omega}_b$ = angular velocity of $\{\underline{\hat{b}}\}$ frame

$\omega_{bi}$ = $\underline{\hat{b}}_i$ components of $\underline{\omega}_b$

$\underline{\omega}_f$ = angular velocity of local deck frame $\{\underline{\hat{f}}\}$

$\omega_{fi}$ = $\underline{\hat{f}}_i$ components of $\underline{\omega}_f$

$\omega_n$ = natural frequency of train axis controller

$\underline{\omega}_p$ = angular velocity of $\{\underline{\hat{p}}\}$

$\omega_P$ = frequency of ship's pitching motion

$\omega_{pi}$ = $\underline{\hat{p}}_i$ components of $\underline{\omega}_p$

$\omega_R$ = frequency of ship's rolling motion

$\omega_T$ = $\dot{\psi}_A$ = rate of change of ship's heading

$\underline{\Omega}_a$ = angular velocity of 'A' rotor

$\Omega_a$ = constant 'A' rotor spin speed

$\underline{\Omega}_b$ = angular velocity of 'B' rotor

$\Omega_b$ = constant 'B' rotor spin speed

$\zeta$ = train axis control damping factor

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## TERMS IN SOLUTIONS

| TERM | WHERE DEFINED |
|------|---------------|
| $d_i$ ($i = 5,6,7,8,11,12,20,21,22$) | Appendix F.6 |
| $d_i$ ($i = 10,13$) | Appendix G |
| $D_i$ | Appendix E |
| $J_{ai}$ | Section 5.0 |
| $J_{bi}$ | Section 5.0 |
| $r_i$ | Appendix 5.F |
| $S_i$ | Appendix F.5 |
| $t_i$ | Appendix C |
| $U_{a3}$ | Appendix F.2 |
| $U_{b1}$ | Appendix F.2 |
| $Y_i$ | Appendix E |

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report presents a rigid body analysis of the dynamics of a shipboard antenna in a three-axis gimbal configuration utilizing Direct Mechanical Stabilization (DMS) for primary control. System parameters are defined and Euler equations applied to obtain a non-linear system model that includes expected torque inputs. A computer simulation is generated with ship input motions due to roll, pitch, turn and flexure and a numerical integration procedure is then employed to determine the motion of the antenna and the resulting pointing error for various scenarios.

The results of the cases studied indicate potential feasibility of this approach for applications requiring pointing accuracy of approximately ±0.15 degree.

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